SOCRATES MINERVA Programme 99843-CP-1-2002-1-PL

Irradiance versus distance. Exploring the inverse square law.

OBJECTIVE

To examine the distance distribution of the visual radiation emitted by the light sources. The irradiance vs. distance dependence is analyzed in terms of the inverse square law.

CONTENTS

THEORY

A. Basic concepts of Radiometry and Photometry

The description and measurements of the propagation of the energy of the electromagnetic waves radiated by light sources uses twofold approaches.
The so called radiometric (physical) approach is based on the entire radiant power produced by the source while the photometric (physophysical) refers only to the part of the radiant power perceived by the human eye as light and human average response function. The spectrum of radiant energy waves that we call light is narrow, ranging from approximately 300nm to 750nm. Wavelengths shorter (UltraViolet) or longer (InfraRed) than these do not produce the visual response in the eye.

Consequently we have two set of quantities and units used in measurements – see table 1. The symbols for radiometric quantities are analogous to the photometric counterpart.

All photometric quantities can be obtained from the their corresponding radiometric quantities on the basis of the spectral luminous efficacy human eye response

The relationship between the (physical) radiometric and the (psychophysical) photometric set units is established by the chosen value of spectral luminous efficacy for human vision: 683 lumens/watt.

In both approaches the basic concepts are similar and describe:

Power of the source

The light source is characterized by the total light output of a light source - total power radiated by it [in watts]. This is called **Radiant power** or **Radiant flux Φ**. In radiometry the unit of **Φ** is **watt [W]**. When referred to the sensitivity of the human eye and so called optical part of the radiated spectrum it determines the Luminous flux (or luminous power). The photometric unit of **Φ** is **lumen [lm].**

Intensity

The commonly used term intensity of light ("brightness") refers to the energy radiated by the source into the unit solid angle in a unit time. This power per unit solid angle is called **Radiant intensity** ***I*** in watts/steradian. The photometric unit of the intensity (**pointance, luminous intensity**) is candela [cd]. The **radiant intensity** enables to describe spatial characteristics of the source. In general, it is given by: $I(\Omega)= d\Phi/d\Omega$.

Illumination

In everyday life our visual reception is based on the effect produced by the light source at the surface of the surrounding objects. This means that we are interested in the amount of radiated energy, which reaches the observed surface element in a unit time. The **irradiance E** expresses radiation power (flux) received by the unit area of the illuminated surface. The unit of irradiance is **watt/m²**. The respective photometric term is called **illuminance** and is given in **lux = lumen/m²**.

The irradiance (as well as the illuminance) of a surface due the point light source depends on the radiant intensity, the distance to the surface, and the orientation of the surface with respect to the source:

$$E = \frac{radiant\ \ flux\,\Phi}{area\ A} = \frac{I\cdot\Omega}{A} = \frac{I\cdot A}{A\cdot r^2} = \frac{I}{r^2} \qquad (1)$$

where flux in a defined solid angle is:

$$\Phi = I\cdot\Omega \qquad (2)$$

and solid angle:

$$\Omega = \frac{A}{r^2} \qquad (3)$$

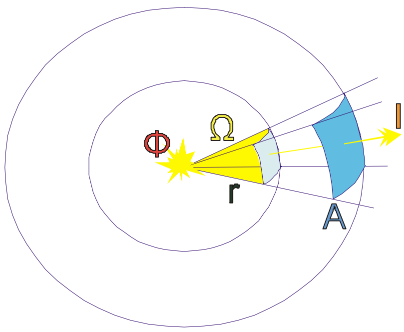

Fig.1. Radiant flux ***Φ*** and radiant intensity ***I***

Solid Angle:

The solid angle **Ω** refers to the cone cut out from the sphere. Solid angle is related to the area A intercepted by the cone on the surface of a sphere of radius r centered on the cone vertex: A/r^2
The unit of the solid angle is **steradian [sr]**.

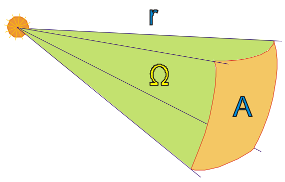

Fig.2. Solid angle

For the surfaces not perpendicular to the direction of light propagation the illumination expression E should be modified in order to deal with the effective (projected) area of the illuminated surface – see the fig.(1):

$$E = \frac{I}{r^2}\cos\phi \qquad (4)$$

where φ is the angle between the normal to the surface and direction of the propagation of the light energy.

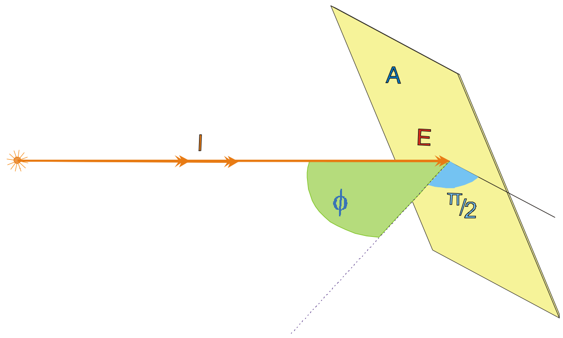

Fig. 3. Irradiance ***E*** at the inclined plane

The inverse square law.

The illumination produced by the point light source, which radiates equally in all directions follows the so-called Inverse square law.
It expresses the fact that amount of energy that passes through the unit area drops with the distance from the source. The total power irradiated by of the source (radiant flux) in all directions (into the full solid angle) remains constant while the total area of the sphere increases with the square of the radius.

$$E \cong \frac{I}{r^2} \qquad (5)$$

So, the power per unit area drops – compare fig. 1. The general character of the inverse square law applies to many other phenomena based on point sources e.g. gravity pf point masses, electrostatic field of point charges etc.

Table 1. Radiometric and photometric quantities.

Radiometry			**Photometry**		
Quantity	**symbol**	**units**	**units**	**symbol**	**Quantity**
Radiant Energy	Q	J	lm s	Q	Luminous energy
Radiant Flux	Φ	W	lm	Φ	Luminous Flux
Irradiance	*E*	W/m^2	lm/m^2	*E*	Illuminance
Radiant Intensity	*I*	W/sr	lm/sr	*I*	Luminous Intensity
Radiance	*L*	$W/(m^2\ sr)$	$lm/(m^2\ sr)$	*L*	Luminance

The radiometric measurements can be performed using the device that allows one to change the distance and the relative angle at which detector views the source (goniometer) – see fig.4. The light detector measures the irradiance.

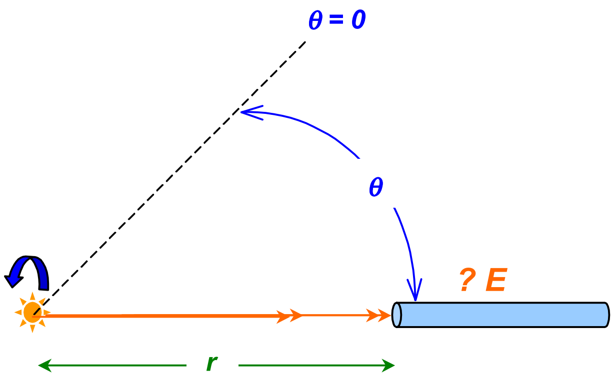

Fig.4. The goniometer

B. <u>Exploring the irradiance vs distance.</u>

The irradiance versus distance dependence can be examined by simultaneously taken measurements of the distance from the detector to the source – *r* and irradiance *E* at the constant angular position ***θ***=0 of the source. The device configuration assumes a point source. Therefore the observation can be analyzed in terms of the inverse square law:

$$E \cong \frac{I}{r^2}$$

The typical dependence is presented on the fig. 5.

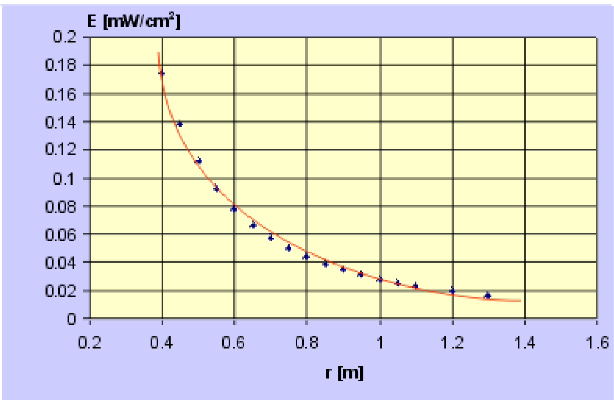

Fig. 5. Illustration of the inverse square law.

<u>APPARATUS FOR THE EXPERIMENTAL EXPLORATION.</u>

Determining the irradiance dependence on the distance can be performed using the simple goniometer consisting of standard lamp handle placed on the optical bench with adjustable light detector holder see fig. 6

The light sensor uses a phototransistor with an active area positioned at a fixed distance from a light source. The detector measures relative irradiance ***E***. The units of irradiance are milliwatts per square centimeter. The light sensor's output is a voltage that is linearly proportional to the amount of irradiance it senses.

This configuration assumes a point source, therefore the inverse square law holds true.

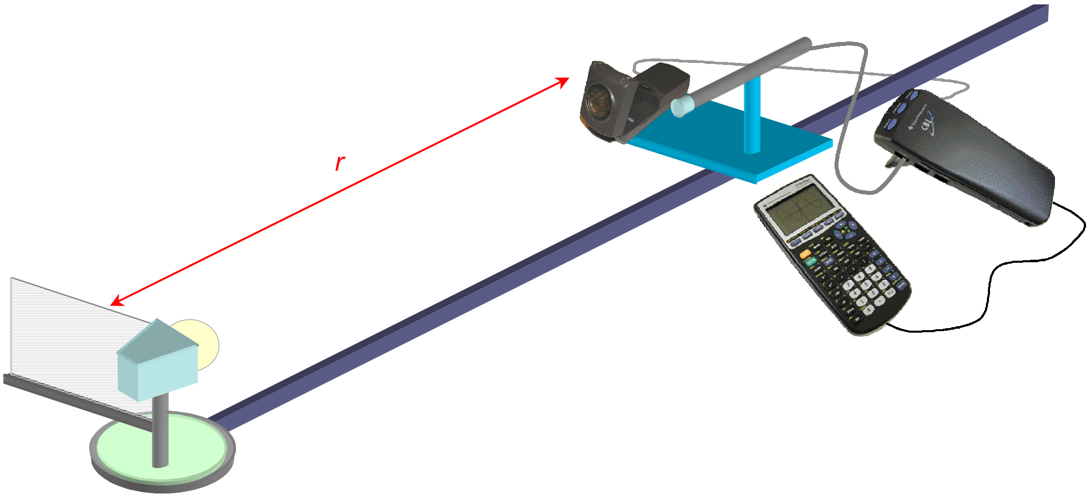

Fig.6. Diagram of the experimental setup

Experiment uses:

i. goniometer
ii. lamp housing E14
iii. incandescent lamp (e.g. 15 - 40W)
iv. Calculator Based Laboratory unit CBL
http://www.vernier.com/legacy/cbl/index.html
or CBL2
http://education.ti.com/us/product/tech/datacollection/features/cbl2.html
v. Light probe (standard CBL or Vernier's LS BTA)
http://www.vernier.com/probes/probes.html?ls-bta&template=standard.html
vi. Calculator Based Ranger CBR (sonic motion detector)
vii. Graphic calculator TI83, TI83 Plus, TI 83 Plus SE.
viii. unit-to unit cable (standard).
ix. CBR – CBL signal cable
x. Program: PHOT2, available for download at: http://www.lepla.edu.pl/
xi. TI-GRAPH LINK TM (optional) cable
http://education.ti.com/us/product/accessory/connectivity/features/cables.html#serialwin
and software
http://education.ti.com/us/product/accessory/connectivity/down/downgraph.html
xii. Personal computer with TI Connect TM software (optional)
Description:
http://education.ti.com/us/product/accessory/connectivity/features/software.html
Download:
http://education.ti.com/us/product/accessory/connectivity/down/download.html

Practical notes about the apparatus setup.

- The Light probe should be connected to the channel CH1 of the CBL unit.
- The CBR sonic detector should be connected to the dedicated SONIC channel of the CBL
- The additional planar sheet should bed mounted to the light source holder and adjusted in the plane of the light source. It reflects the ultrasonic wave of CBR and helps measuring the distance.
- The CBR detector should be placed on the same stage as the light detector and adjusted with the active end of it. So, the measured distance represents distance from the source.
- During the measurements the detector/CBR stage should not be placed closer than 0.4 m from the light source.
- One should adjust the height of the detector's mount with the center of the light source.
- The ambient light should be reduced and kept stable during the experimental session.

DATA ACQUISITION (TI 83)

In the experiment the light sensor measures relative irradiance ***E*** in milliwatts per square centimeter as a function of the distance ***r*** from the source. The distance is being changed by slowly sliding the detector stand with respect to the source holder. Data will be presented in the rectangular coordinates as the E(r) plot.
The experiment is controlled by means of the preloaded calculator program PHOT2.

Experimental procedure is divided into the preparatory part and the data acquisition.

Preparation:

Make all connections as shown on the Fig.6.
Turn on the calculator, the CBL and prepare CBR.

Measuring the ambient light should precede the main measurements.

1. Launch the program PHOT2 by choosing its name from the NPEK menu.
2. Choose ZERO PROBE from the program menu – Fig.7.
3. Keep the light source off and expose light sensor to the background. Start collecting data – Fig.8.

Fig.7.

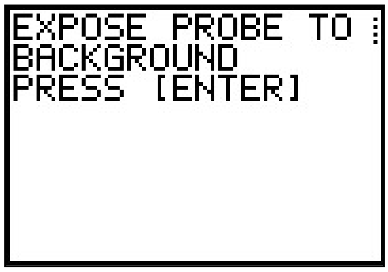

Fig.8.

The collected irradiance value will be subtracted from any other collected irradiance data.

The described calibration is to be done once during the experimental session provided that the background illumination does not change.

Data collection

1. Adjust the detector/CBR stage at a distance 0.5m from the source. Check position of the light detector. Its axis should exactly match the source.
2. Choose option 1: COLLECT DATA from the main menu.
3. Turn on the light source.
4. Pres [ENTER] key on the calculator and start sliding the detector stage away from the source. Adjust speed of that motion so, that the whole displacement (approx. 1m) takes 20 sec. – Fig.9.
5. Data collection ends and the irradiance vs distance plot is displayed – fig. 10.
6. If you are not satisfied with he obtained data, repeated using the same or new settings (new calibration is not necessary). Choose 1:REPEAT from the CONTROL menu – Fig.11.
7. Stop measurements choosing 2: RETURN TO MAIN and then 2: QUIT from main menu – Fig. 7.
8. Collected data are stored in a calculator's memory and you can proceed using the standard calculator's functions. Now you can disconnect the CBL from the calculator.

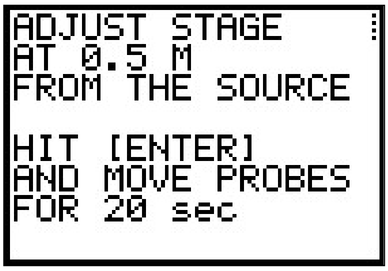

Fig. 9

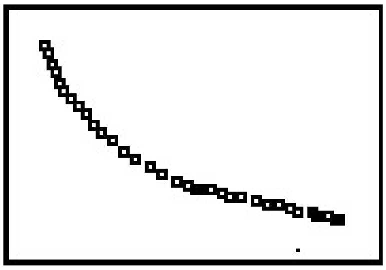

Fig.10

Fig.11

DATA ANALYSIS (TI83)

Further analysis can be performed using tools implemented in calculators (or other analytical software tools such as MS Excel spreadsheet).

Collected data are stored in the calculator's lists:
- Time in sec - List L_1
- Distance in m – List L_2
- irradiance ***E*** in watts/cm^2 - List L_3

Exemplary data are available for a download in the following calculator type files:

- distance as the file - Data sample/TI83/ $\mathbf{L_2}$
- irradiance - Data sample/TI83/ $\mathbf{L_3}$

Experimental data plot is defined as Plot1 (L_3 vs. L_2) and can be recalled by STAT PLOT menu.

Analysis of plots

Analysis of the obtained distance dependence of irradiance can be done by graphical exploration of the plots.
The analysis can be performed in terms of the inverse square law (eq. 5):

$$E \cong \frac{I}{r^2}$$

The equation states that the irradiance is linearly proportional to the inverse of the distance square. One may examine this statement in case of the collected data by making the respective plots. This can be done in two ways.

Linearization

According to the eq.5. the ***E(1/r²)*** plot should have a form of a straight line. In order to examine this the original distance data gathered in the list L_2 - r, should be transformed to $1/r^2$. The transformed data are stored in a new list L_4 – Fig.12.
Now, one the new plot should be defined as L_4 vs. L_3 and displayed – Fig. 13. The data points well match the straight line, so assumption of the linear dependence is justified and one can apply the linear regression model. The linear regression is recalled from the STAT CALC menu (Fig.14) with the respective arguments – Fig. 15. As the result the parameters of the linear function are calculated and the function is stored as the Y_1. The result of the regression is displayed – Fig. 16.
The correlation coefficient ***r*** expresses the quality of the approximation. In a given example the ***r*** value is close to 1, which confirms good linear correlation between irradiance ***E*** and ***1/r²***.
This can be seen as well in the combined plot of experimental data and regression line – Fig. 17.

Fig.12

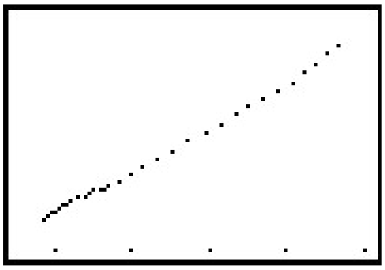

Fig.13

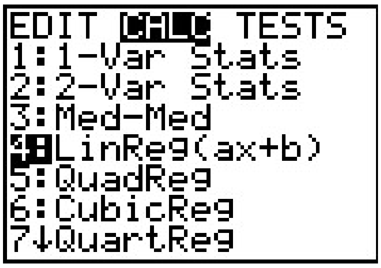

Fig.14

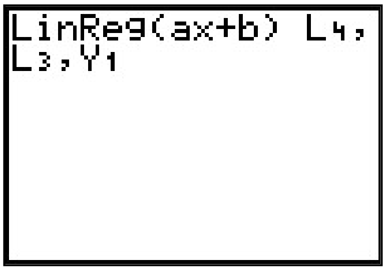

Fig.15

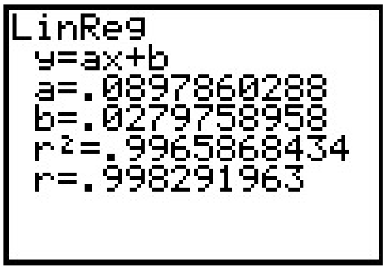

Fig.16

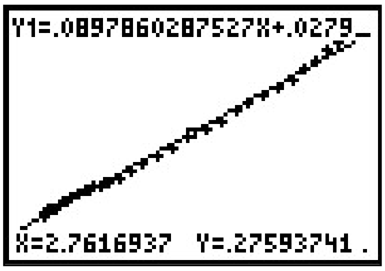

Fig.17

Power regression

The ***E(1/r²)*** expression can be validated by direct application of the power regression model to the original irradiance (distance) data.
One can select the PwrReg model from the STAT CALC menu and provide the list of respective arguments – Fig. 18.

Fig.18

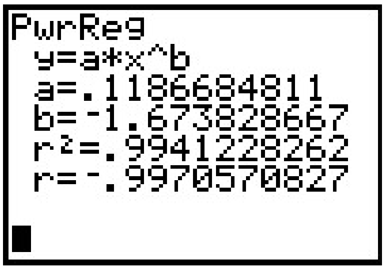

Fig.19

As the result the parameters ***a*** and ***b*** of the power function ***y=ax^b*** are calculated and the function is stored as the Y_2. The result of the regression is displayed – Fig. 19.
The correlation coefficient ***r*** expresses the quality of the approximation. In a given example the ***r*** value is close to 1, which confirms good power correlation between irradiance ***E*** and ***r***.
In the given example the obtained exponent ***b*** is close to –2. This results states that the observed irradiance vs distance dependence follows the inverse square law.
This can be seen as well in the combined plot of experimental data (Plot1 – Fig.20) and regression curve – Fig. 21.

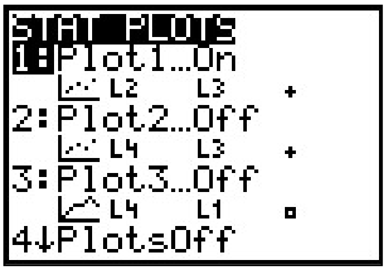

Fig.20

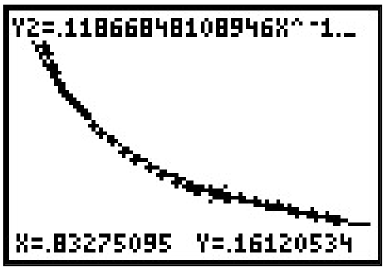

Fig.21

<u>DATA ANALYSIS (MS EXCEL)</u>

Analysis can be performed using tools implemented in calculators and personal computer functions offered by the MS Excel software.

Transfer of experimental data to the personal computer.

After completing the experiment data could be transferred from the graphic calculator to the personal computer.
TI GRAPH LINK™ cable supported by the TI Connect™ software offer tools enabling exploration of the contents of calculator (TI DEVICE EXPLORER) and data edition (TI DATA EDITOR).
Data collected in the experiment are stored in the calculator's lists:

- distance in meters - List L_2
- irradiance mW/cm^2 - List L_3

Within the TI Connect™ program - the TI DEVICE EXPLORER one can save calculator's lists on the computer's hard disk and then open them within TI DATA EDITOR. Option **Special Lists Export** provides opportunity to save the chosen list as the Excel Comma Separated type file (*CSV file). Such a file could be open and explore within the MS Excel™ spreadsheet software.

Exemplary data are available for a download in the following files:

- angle: as file - Data sample/MSExcel /**dist**
- irradiance: as file - Data sample/MSExcel/**intensity**

Analysis of plots

Create a spreadsheet and import the data from the files **angle** and **intensity.** Make the plot E(r) from the raw experimental data. Choose scattered type of the plot – Fig. 22.

The analysis can be performed in terms of the inverse square law (eq. 5):

$$E \cong \frac{I}{r^2}$$

The equation states that the irradiance is linearly proportional to the inverse of the distance square.
One may examine this statement in case of the collected data by making the respective plots.
This can be done in two ways.

Linearization

According to the eq.5. the ***E(1/r²)*** plot should have a form of a straight line. In order to examine this the original distance data r, should be transformed to $1/r^2$. The transformed data are stored in a new column of the spreadsheet.

Now, one the new plot should be defined and displayed. The data points well match the straight line, so assumption of the linear dependence is justified and one can apply the linear regression model by adding trend line (linear type) to the plot. The trend line is drawn and resulting linear expression is displayed in the plot – Fig. 23.The determination coefficient R^2 expresses the quality of the approximation. In a given example the ***r*** value is close to 1, which confirms good linear correlation between irradiance ***E*** and ***1/r²***.

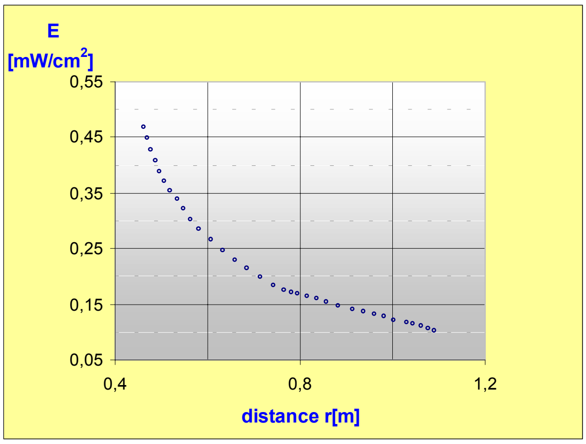

Fig.22. Irradiance vs distance. Experimental data

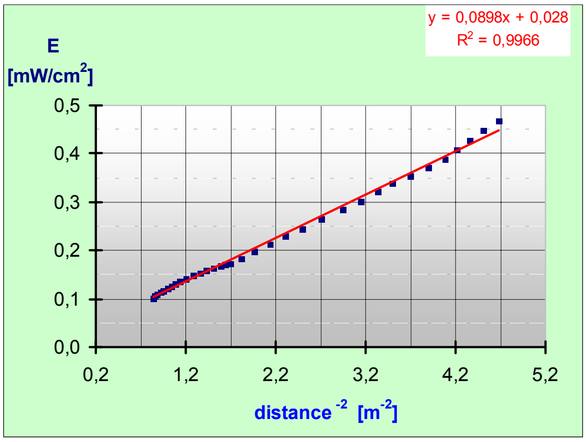

Fig.23. Irradiance ***E*** vs ***1/r²*** dependence. Linear regression.

Power regression

The ***E(1/r²)*** expression can be validated by direct application of the power regression model to the original irradiance (distance) data.
This can be done by applying the power type ($y=ax^b$) of the trend line added to the original ***E (r)*** plot.
The trend line is drawn and resulting power expression is displayed in the plot – Fig. 24.
The determination coefficient R^2 expresses the quality of the approximation. In a given example the R^2 value is close to 1, which confirms good power type correlation between irradiance ***E*** and ***r***.

In the given example the obtained exponent ***b*** is close to –2. This results states that the observed irradiance vs distance dependence follows the inverse square law.

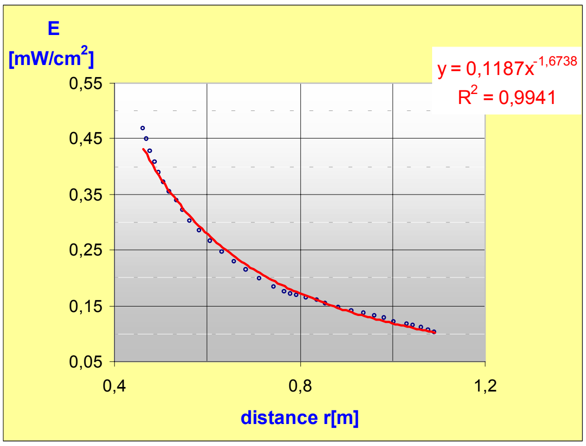

Fig.24. Irradiance vs distance. Experimental data and power type approximation

Note:

The complete numerical analysis is presented in the MSExcel file:
Data sample/MSExcel /invsqranalysis