Radioactive decay: modelling in Excel

www.lepla.eu

SOCRATES MINERVA Programme
99843-CP-1-2002-1-PL

A simple example of exponential decay in physics is radioactive decay. Exactly when a particular radioactive nuclei will decay cannot be predicted, but for a given material we can say that an individual nuclei has a probability λ (called the decay constant) of decaying in unit time.

Content:

- Theory of Exponential Decay
- Half-life
- Solution in Excel

Theory of Exponential Decay

A simple example of exponential decay in physics is radioactive decay. Exactly when a particular radioactive nucleus will decay cannot be predicted, but for a given material we can say that an individual nucleus has a probability λ (called the decay constant) of decaying in unit time. Thus if we have *N* radioactive nuclei at time *t* then we expect the change in the number of nuclei, *dN*, in a short time *dt* to be given by

$$dN = -\lambda \cdot N dt$$

where the negative sign indicates that the number of radioactive nuclei is decreasing. We can rewrite the above as a differential equation:

$$\frac{dN}{dt} = -\lambda \cdot N$$

Hence if there are N_0 radioactive nuclei present at the start of a period of observation (time *t* = 0) and *N* radioactive nuclei present at time *t*:

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$

Remembering that the integral of *1/x* is the natural logarithm of *x* yields:

$$\ln(N) - \ln(N_0) = -\lambda \cdot t$$

Now from the laws of logarithms *ln(A/B) = ln(A) - ln(B),* so:

$$\ln\left(\frac{N}{N_0}\right) = -\lambda \cdot t$$

Thus:

$$N = N_0 \cdot e^{-\lambda t}$$

Both N and N_0 are numbers of atoms and so are dimensionless, whilst the dimension of t is seconds, hence λ must have a dimension of s^{-1}.

Half life

The half-life, $T_{1/2}$, is the time required for the number of radioactive nuclei to decrease to one-half of the initial value N_0 to obtain an expression for the half-life we set $N = \frac{1}{2}N_0$ and set $t = T_{1/2}$ in equation 1. Cancelling the common term of N_0 and taking logarithms gives

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

Radioactive nuclei are usually specified in terms of their half-life, rather than the value of their decay constants.

Example

In an experiment the decay process

$$^{222}Rn \rightarrow {}^{218}Po + \alpha$$

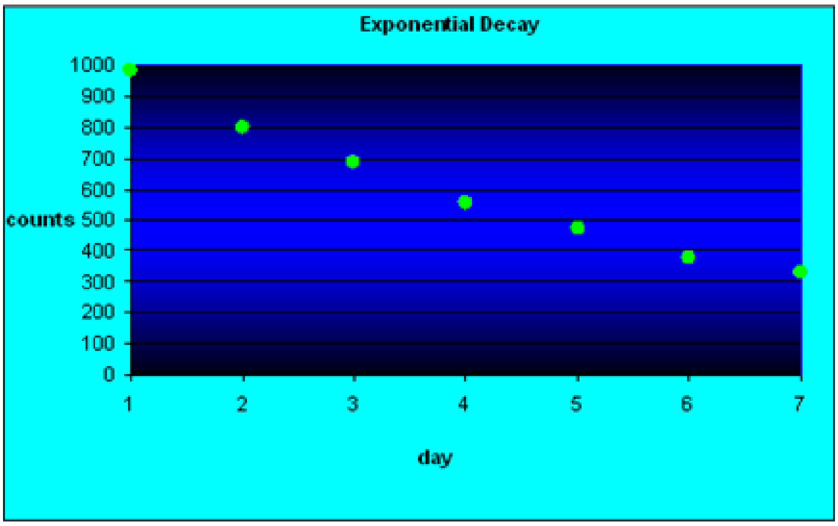
is studied. Activity is monitored every day for a week using a Geiger counter. Counts are taken over a period of 10 minutes. Results are shown below

Day	Geiger counts
1	980
2	800
3	685
4	556
5	471
6	377
7	330

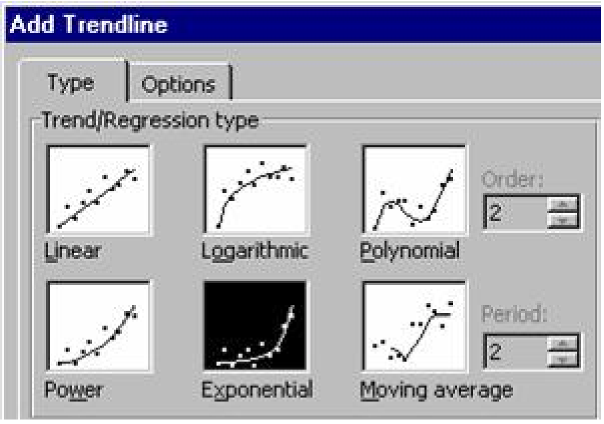
What is the half-life of the decay process?

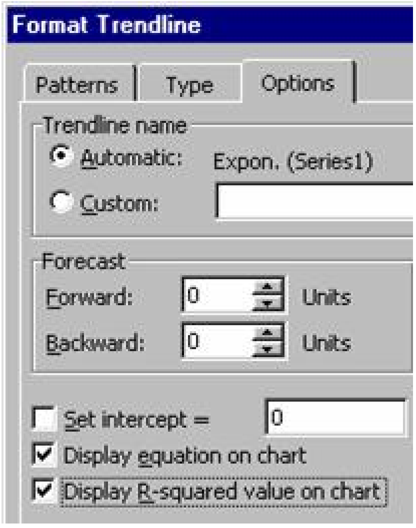
Solution in Excel

Using the techniques learnt in earlier modules we plot the data in Excel:

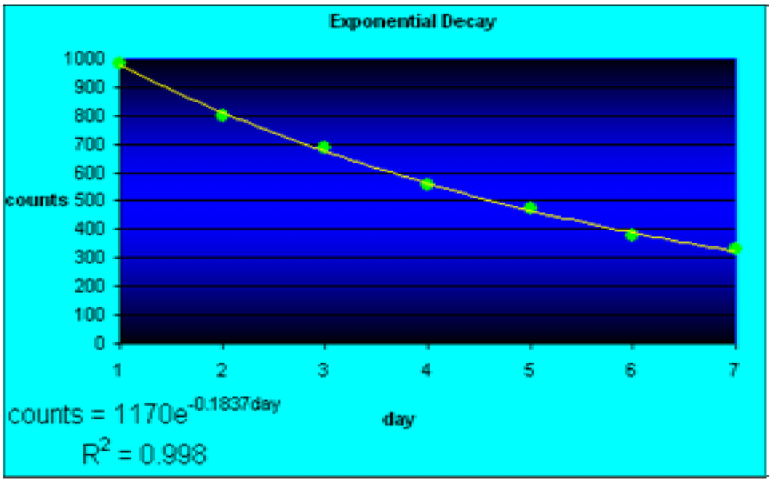


Now add a trendline.





Choose the Exponential option and display both the equation (with appropriate variables) and the R^2 value on the chart.



We see that this is almost a perfect fit.

To determine the half-life, use equation (2) i.e. $T_{\frac{1}{2}} = \dfrac{\ln 2}{0.1837} \cong 3.77 days$.