

## Toy car on an incline



The aim of the experiment is to explore the basic features of the motion of a body on an incline.

For this purpose data about position, velocity and acceleration as function of time of a toy car going up and down an incline will be collected and analysed.

**This file contains the pages of the web site<sup>1</sup> in this order:**

THEORETICAL MODEL

DATA SAMPLE

*EXPERIMENT AND DATA ANALYSIS WITH TI83*

*DATA ANALYSIS WITH MS EXCEL*

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<sup>1</sup> The links to pages outside this document have been omitted.

## THEORETICAL MODEL

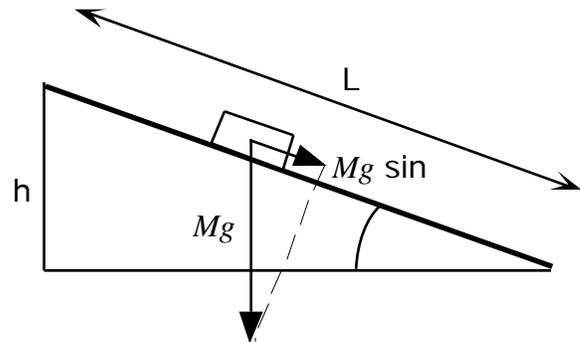
### 1) MOTION WITHOUT FRICTION

Assuming that the cart can be considered as a point mass moving along the incline, the total force acting on it is  $Mg \sin$  and the expected value of acceleration is

$$a_t = g \sin \phi$$

where  $\phi$  is the tilt angle that can be calculated given the plane length  $L$  and height  $h$  using

$$\sin \phi = h / L$$



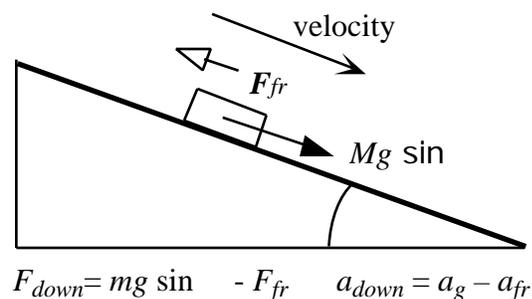
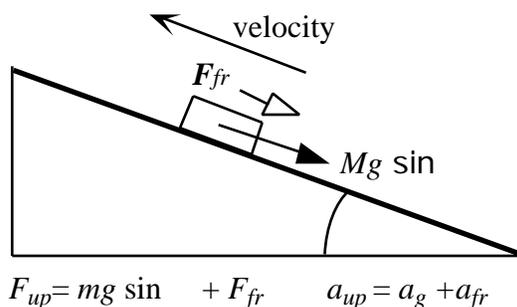
The uncertainty in the expected value depends upon the accuracy in the measurements of  $L$  and  $h$

$$a_t / a_t \simeq \Delta h/h + L/L$$

Special attention should be given to the measurement of  $h$ . A systematic error can arise from assuming - without checking - that the plane of reference is horizontal (see Note below).

### 2) MOTION WITH FRICTION

If friction is not negligible acceleration will be different during the uphill and downhill motion. The friction force is always directed opposite to the velocity of the cart on the incline, therefore it adds to the gravity component when the cart moves upward and subtracts when it moves downward.



*Toy car on an incline*

Therefore:  $g \sin \theta = (a_s + a_d)/2$

$$a_{fr} = (a_s - a_d)/2$$

**NOTE: HOW TO OBTAIN A MORE PRECISE MEASUREMENT OF THE TILT ANGLE**

To check the table horizontality one may use a sphere (diameter > 2 cm), and verify it does not roll on the table.

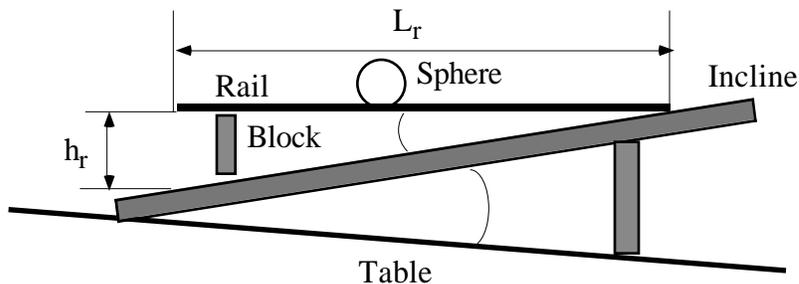
Alternatively one may measure the incline tilt with great accuracy by using the following procedure.

- 1) place an “U” shaped rail (about 1 meter long) on the incline, keeping it approximately in horizontal position by means of a sliding block as shown in the figure;
- 2) place the sphere on the rail, and adjust the block position until the sphere stays in equilibrium;
- 3) measure the rail length  $L_r$  and the distance  $h_r$  between the rail end and the incline.

The tilt of the incline with respect to horizontal plane is given by the equation

$$\sin \theta = h_r / L_r.$$

Note that the measured angle may be quite different from the angle  $\theta$  between the incline and the plane of the supporting table



## DATA SAMPLE

Sample data are downloadable in various formats :

**Graphic Calculator data files (created by TI 83):**

**time:** [time.8XI](#)

**position:** [pos.8XI](#)

**velocity:** [vel.8XI](#)

**acceleration:** [acc.8XI](#)

The files were created by RANGER. They can be opened by TI-Connect software (to read the values) and can be copied from your PC into a Graphic Calculator, using the TI-GraphLink cable (gray, black or UBS).

**Graphic Calculator data file (created by TI 89):** **incline** [download](#)

The file was created by SCIENCE. It can be opened by TI-Connect software (to read the values) and can be copied from your PC into a Graphic Calculator, using the TI-GraphLink cable (gray, black or UBS).

**MS-Excel data file:**

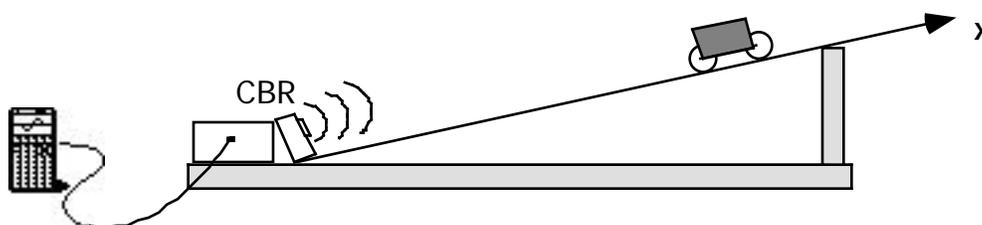
**TOY.xls** [download](#)

## **EXPERIMENT AND DATA ANALYSIS WITH TI83**

### **EXPERIMENTAL SETUP AND PROCEDURE (TI83)**

In order to perform this experiment the following items are needed:

- Incline (made up, for example, by a shelf from a bookcase or an inclined plane from the physics lab)
- Toy car (or small cart with light wheels)
- Graphing calculator TI-83plus
- Motion sensor Ranger (located at the lower end of the incline)
- Black cable to connect graphing calculator to Ranger



Before starting the experiment you need to check that the program Ranger has been installed in the calculator by scrolling through the programs list. If necessary the program can be downloaded from the Ranger itself (see Instruction manual).

Now proceed through the following steps

- connect Ranger to calculator using the black cable (make sure that cable is firmly connected to both)
- set up calculator for data acquisition and display of position vs. time graph (see next section)

push the car up the incline and start acquisition.

### **CALCULATOR SETUP AND DATA ACQUISITION (TI83)**

From the MAIN MENU select 1:SETUP/SAMPLE.

Choose setup as follows:

REALTIME:NO  
TIME(S):2  
DISPLAY: DIST  
BEGIN ON: ENTER  
SMOOTHING: NONE  
UNITS: METERS

With this setup data will be collected for 2 seconds and the plot of position vs. time will be displayed at the end of data collection. After a first run you may notice that, for your particular experimental setup, a different choice for the total time of the experiment would be more suitable in which case you may decide to change it in the SETUP/SAMPLE Menu.

Now select START NOW and press ENTER to start acquisition.

At the end the data will be transferred from the Ranger to the calculator and the position vs. time graph will appear on the screen.

## **GUIDE TO DATA ANALYSIS (TI83)**

The following steps are suggested in order to analyse the collected data.

### **1) Explore the position, velocity and acceleration plots**

- Examine the position, velocity and acceleration plots and, comparing one with the other, try to answer to the questions:
  - Can you detect the interval corresponding to uphill and downhill motion in the position plot? And in the velocity plot?
  - In the velocity plot can you identify an almost straight portion of the graph? what is the meaning of its slope? Why is the slope negative?

### **2) Determine the value of acceleration**

- Calculate the value of acceleration during uphill and downhill motion by linear regression on velocity data

In order to find out the acceleration value you need to select the corresponding portion of the velocity plot (the approximately straight line with negative slope) and to apply a linear regression to the selected data.

A finer analysis of the velocity plot may suggest that acceleration has a different value when the car is moving up from when it is moving down. In that case the two different portions of the plot should be selected separately and two separate regressions should be performed. The mean value of the two results should correspond to the component of gravity acceleration parallel to the direction of motion (see Theoretical Model).

### **3) Comparing theoretical and experimental values of acceleration**

Assuming that the body behaves like a point mass moving on the incline, we can calculate the theoretical expected value of acceleration and compare it with the experimental value.

Are the two values compatible? If not, what could be the reason for this difference?

In this guide we suggested to calculate the acceleration by performing a linear regression on the velocity plot. This choice is usually convenient because the portion of graph to be selected is more easily identified.

You can also try to calculate the acceleration value from the position plot (quadratic regression) or from the acceleration one (mean value on the same time interval). The values should be consistent with one another.

An example of analysis performed on the downloadable “Sample Data” is described below.

### COMPLETE DATA ANALYSIS -TI83 (1)

You are expected to obtain plots like those shown below.



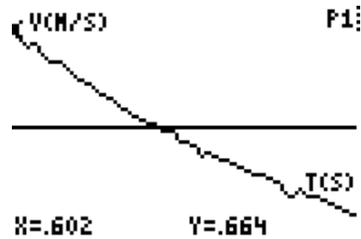
The central part of the position plot is not symmetrical thus revealing that acceleration has not the same value while the toy car is going uphill and downhill. Still it may be important to perform a linear regression selecting all the data at a time to stress that as a first approximation the acceleration value should be the same.

Below are the steps to select the data:

```

MAIN MENU      :      PLOT MENU      :      PLOT TOOLS      :
1: SETUP/SAMPLE 1: DIST-TIME      1: SELECT DOMAIN
2: SET DEFAULTS 2: VEL-TIME        2: SMOOTH DATA
3: APPLICATIONS 3: ACCEL-TIME      3: PLOT MENU
4: PLOT MENU     4: PLOT TOOLS
5: TOOLS         5: REPEAT SAMPLE
6: QUIT         6: MAIN MENU
                7: QUIT
    
```

On the screen you then are asked to select first the lower and then the upper bound; this will give you a plot like the one below:



In order to perform a linear regression one must exit the RANGER program and use the regression tools of the TI-83.

By pressing STAT and then selecting CALC a menu appears from which you can choose 4:LinReg(ax+b). By inserting L1 as X and L3 as Y you obtain for the sample data:

```

LinReg
y=ax+b
a=-1.260331277
b=1.315036181
a = - 1.26 m/s2

LinReg(ax+b) L1,
L3

```

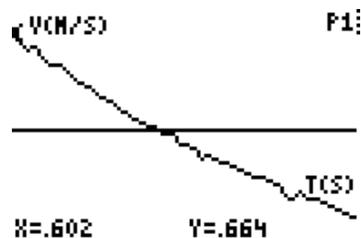
This value is to be compared with the theoretically expected one. In the case of this experiment we had  $\sin \theta = h/L = 0.133$  with about 1% uncertainty. Therefore the expected value is

$$a_t = g \sin \theta = - (1.30 \pm 0.01) \text{ m/s}^2$$

which is not in complete agreement with the experimental one.

### COMPLETE DATA ANALYSIS –TI83 (2)

A closer inspection of the velocity plot may reveal, as mentioned before, that there are two different lines joined at the instant when the body has zero velocity.



The two different slopes can be calculated by selecting the two time intervals separately. In order to repeat the selection twice you should retrieve the original data from CBR (or from the original lists if you copied them under different names and kept them) by following these steps:

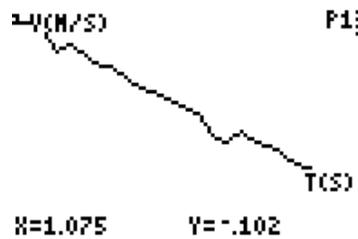
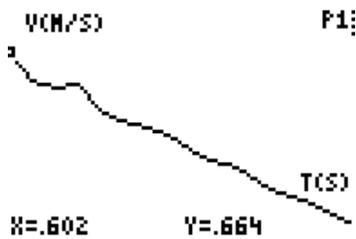
```

MAIN MENU
1: SETUP/SAMPLE
2: SET DEFAULTS
3: APPLICATIONS
4: PLOT MENU
5: TOOLS
6: QUIT
    
```

```

TOOLS
1: GET CBR DATA
2: GET T183 DATA
3: CBR STATUS
4: STOP/CLEAR CBR
5: MAIN MENU
    
```

With the sample data the two selected portions of the velocity graph and the corresponding two values for acceleration are the following:



```

LinReg
y=ax+b
a=-1.625999472
b=1.613177176
    
```

```

LinReg
y=ax+b
a=-.9953910555
b=.9640031457
    
```

$$a_{up} = -1.63 \text{ m/s}^2$$

$$a_{down} = -0.99 \text{ m/s}^2$$

The mean value (i.e. the component of acceleration due to gravity) is therefore

$$a_m = (a_{up} + a_{down}) / 2 = (a_g + a_{fr} + a_g - a_{fr}) / 2 = a_g = -1.31 \text{ m/s}^2$$

and a rough estimate of the friction contribution to acceleration can be obtained as

$$a_{fr} = (a_{up} - a_{down}) / 2 = -0.32 \text{ m/s}^2$$

The acceleration value is in agreement with the expected one  $[-(1.30 \pm 0.01) \text{ m/s}^2]$ .

Note that this value would coincide with the one obtained with a single regression of the two sets of data only if the number of data in the two sets were the same. Therefore two separate regressions are advisable in general to guarantee that the same weight is attributed to uphill and downhill contributions.

Note that disagreement between the expected and the measured values of acceleration can be due to:

- 1) Measurement of relative to a plane that is not horizontal (even little deviations can noticeably affect the value of the expected acceleration; see Note to Theoretical Model)
- 2) Total mass of wheels not negligible if compared to the total mass of the toy car (in which case the rotational motion of wheels should be taken into account).

## **DATA ANALYSIS WITH MS-EXCEL**

### **GUIDE TO DATA ANALYSIS with MS Excel**

Data obtained from this experiment can be imported into MS Excel using the TI Graph Link cable and the TI Connect software.

The following steps are suggested in order to analyse the collected data.

#### **1) Explore the position, velocity and acceleration plots**

- \* Construct the position, velocity and acceleration plots.
- \* Examine the position, velocity and acceleration plots and, comparing one with the other, try to answer to the questions:
  - i. Can you detect the interval corresponding to uphill and downhill motion in the position plot? And in the velocity plot?
  - ii. In the velocity plot can you identify an almost straight portion of the graph? what is the meaning of its slope? Why is the slope negative?

#### **2) Determine the value of acceleration**

Calculate the value of acceleration during uphill and downhill motion by linear regression on velocity data.

In order to find out the acceleration value you need to select the corresponding portion of the velocity plot (the approximately straight line with negative slope) and to apply a linear regression to the selected data.

A finer analysis of the velocity plot may suggest that acceleration has a different value when the car is moving up from when it is moving down. In that case the two different portions of the plot should be selected separately and two separate regressions should be performed. The mean value of the two results should correspond to the component of gravity acceleration parallel to the direction of motion (see Theoretical Model).

#### **3) Comparing theoretical and experimental values of acceleration**

Assuming that the body behaves like a point mass moving on the incline, we can calculate the theoretical expected value of acceleration and compare it with the experimental value.

Are the two values compatible? If not, what could be the reason for this difference?

In this guide we suggested to calculate the acceleration by performing a linear regression on the velocity plot. This choice is usually convenient because the portion of graph to be selected is more easily identified.

You can also try to calculate the acceleration value from the position plot (quadratic regression) or from the acceleration one (mean value on the same time interval).

The values should be consistent with one another.

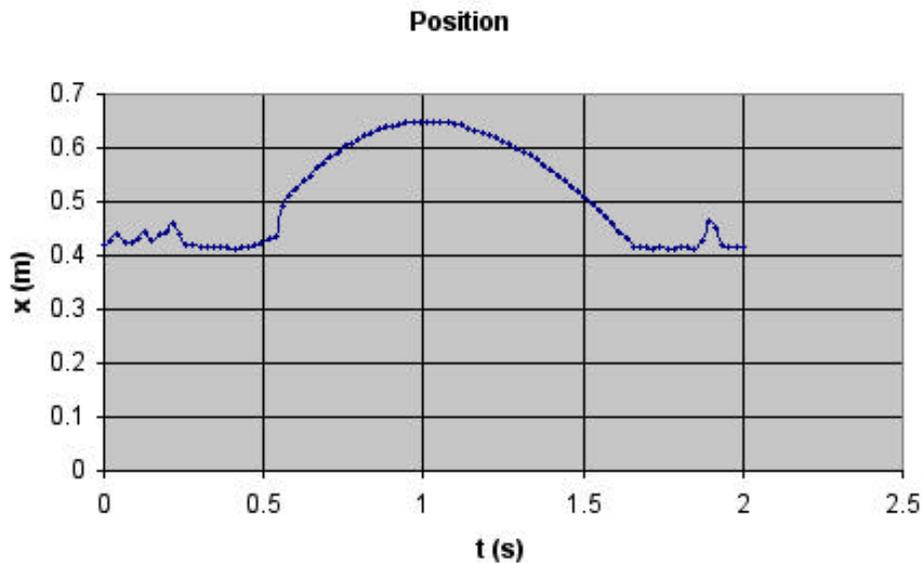
An example of “Complete Analysis – MSEXcel” performed on the downloadable “Sample Data” is available below.

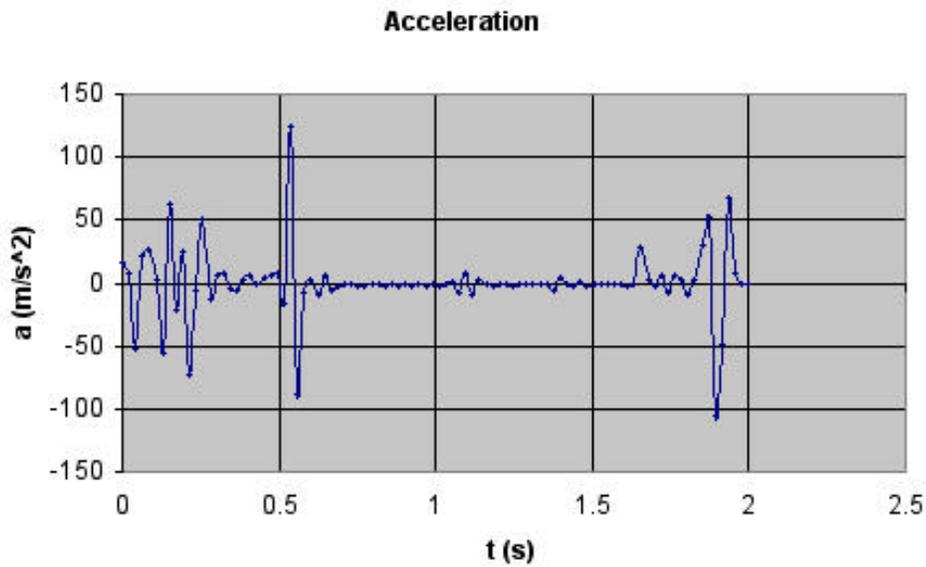
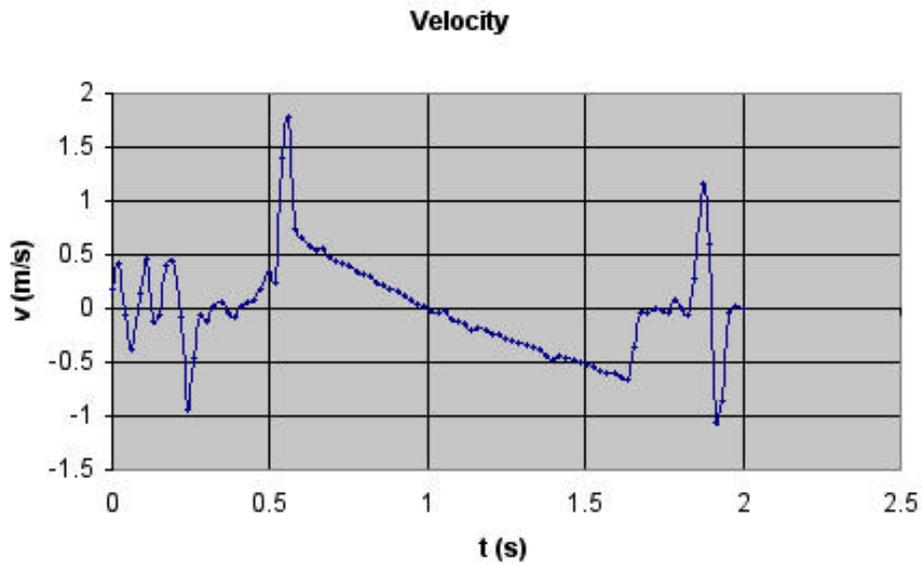
### COMPLETE ANALYSIS - MS-Excel (1)

Motion data from the experiment have been imported into the file [Toy.xls](#) .

Time data (in seconds) are in column A, distance (in meters) in column B, velocity (in m/s) in column C, acceleration (in  $\text{m/s}^2$ ) in column D.

By using these data, you are expected to obtain plots like those shown below.

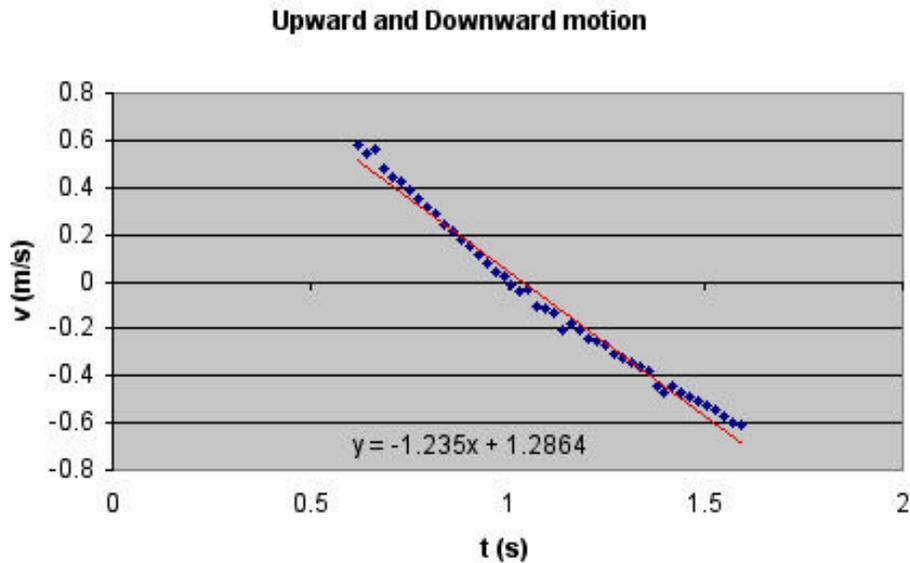




The central part of the position plot is not symmetrical thus revealing that acceleration has not the same value while the toy car is going uphill and downhill. Still it may be important to perform a linear regression selecting all the data at a time to stress that as a first approximation the acceleration value should be the same.

## COMPLETE ANALYSIS - MS-Excel (2)

Select the portion of data concerning the upward and downward motion.  
Now you need to perform a linear regression on the selected data.



The value obtained for acceleration is:

$$a = -1.24 \text{ m/s}^2$$

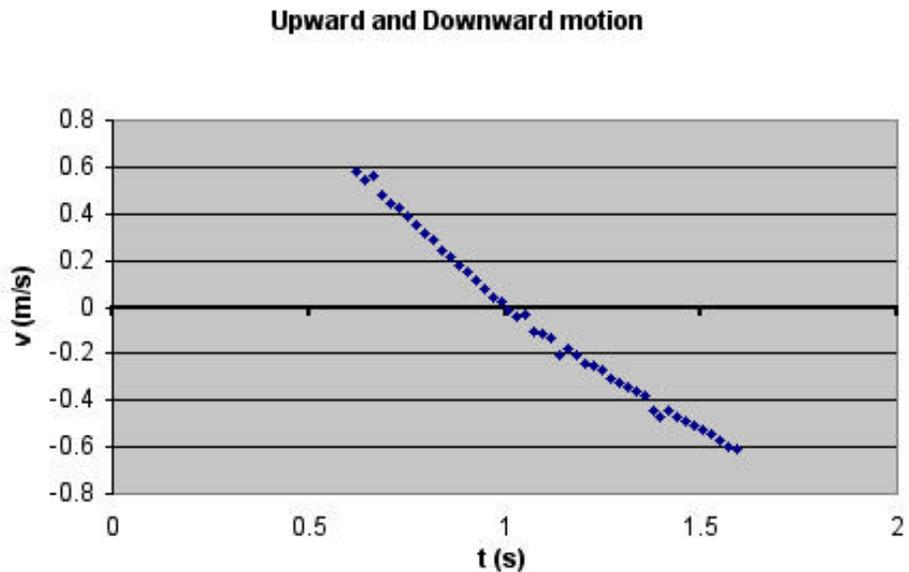
This value is to be compared with the theoretically expected one. In the case of this experiment we had  $\sin \theta = h/L = 0.133$  with about 1% uncertainty. Therefore the expected value is

$$a_t = g \sin \theta = -(1.30 \pm 0.01) \text{ m/s}^2$$

which is not in complete agreement with the experimental one.

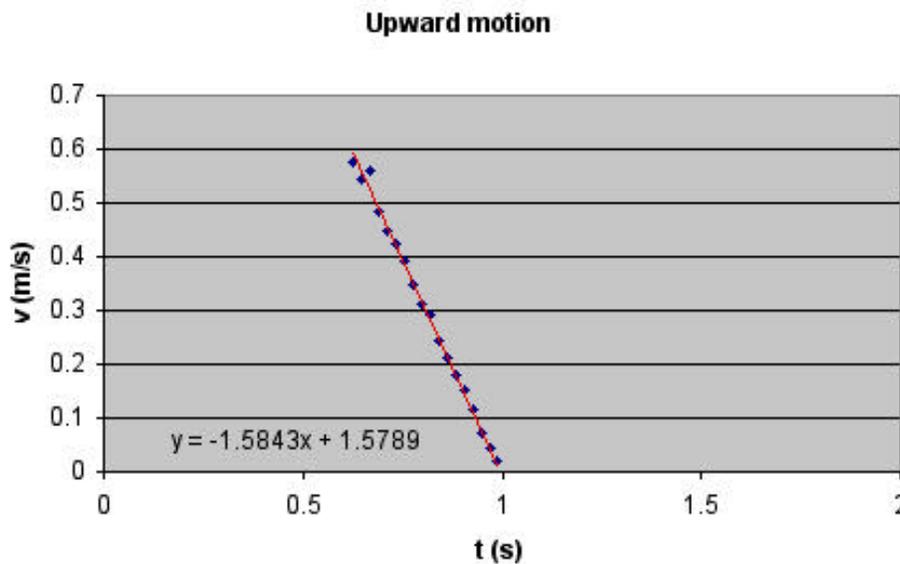
### ANALISI COMPLETA - MS-Excel (3)

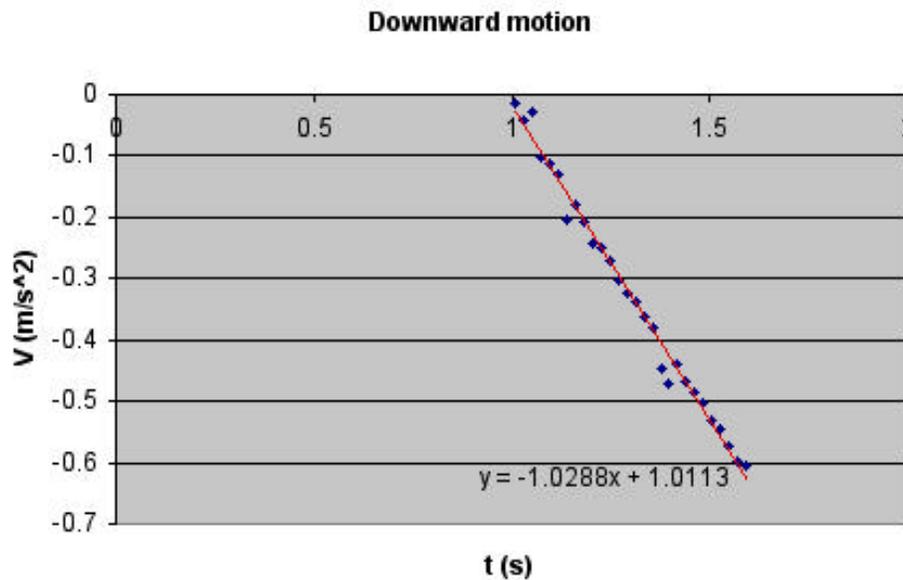
A closer inspection of the velocity plot may reveal, as mentioned before, that there are two different lines joined at the instant when the body has zero velocity.



The two different slopes can be calculated by selecting the two time intervals separately and performing two separate linear regressions.

With the sample data the two selected portions of the velocity graph and the corresponding two values for acceleration are the following:





We therefore obtain the corresponding acceleration values:

$$a_{up} = -1,58 \text{ m/s}^2$$

$$a_{down} = -1,03 \text{ m/s}^2$$

The mean value (i.e. the component of acceleration due to gravity) is therefore

$$a_m = (a_{up} + a_{down}) / 2 = (a_g + a_{fr} + a_g - a_{fr}) / 2 = a_g = -1.31 \text{ m/s}^2$$

and a rough estimate of the friction contribution to acceleration can be obtained as

$$a_{fr} = (a_{up} - a_{down}) / 2 = -0.55 \text{ m/s}^2$$

The acceleration value is in agreement with the expected one  $[-(1.30 \pm 0.01) \text{ m/s}^2]$ .

Note that this value would coincide with the one obtained with a single regression of the two sets of data only if the number of data in the two sets were the same. Therefore two separate regressions are advisable in general to guarantee that the same weight is attributed to uphill and downhill contributions.

Note that disagreement between the expected and the measured values of acceleration can be due to:

- 1) Measurement of  $\theta$  relative to a plane that is not horizontal (even little deviations can noticeably affect the value of the expected acceleration; see Note to Theoretical Model)
- 2) Total mass of wheels not negligible if compared to the total mass of the toy car (in which case the rotational motion of wheels should be taken into account).