TRANSIENTS IN THE RC CIRCUIT

OBJECTIVE

To examine the process of charging the capacitor in the RC circuit..
To determine the time constant of the RC circuit and to calculate the charge of the capacitor.
The process of charging the capacitor in the DC serial connection of R and C is observed by measuring the time dependence of the voltage across the resistor. The U(t) is used for determine the time constant of the RC circuit. The current/time dependence is used for calculation of the charge of the capacitor.

CONTENTS

I.   **Theoretical model.**
II.   **Apparatus for the experimental exploration.**
III.   **Data acquisition.**
IV.   **Data analysis (TI83)**
V.   **Data analysis (MS Excel)**

THEORETICAL MODEL

In the act of charging the capacitor the current, voltages and charge of the capacitor change with time, Figure 1. Shows a basic RC circuit. Initially capacitor is uncharged and then at some initial time $t=0$ closing the switch permits current around the loop to begin charging the capacitor.
Because the capacitor is initially uncharged the potential difference across it is initially $U_C=0$. At this time the voltage across the resistor U_0 is equal to the battery *EMF* and the initial current in the circuit is i_0 .
As the capacitor charges it's voltage U_C increases and the potential difference across the resistor decreases, corresponding to the decrease of the current. After some time the capacitor is fully charged, current decreases to zero and the potential difference across the resistor becomes zero. Then the entire battery *emf* appears across the capacitor.
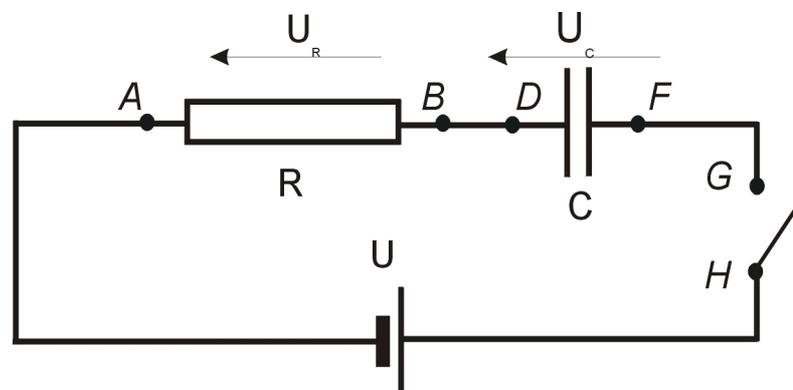

Figure 1. RC circuit

The voltage U_R across the resistor is time dependent and follows the equation:

$$U_R = U_0 \exp(-\frac{t}{RC}) \quad (1)$$

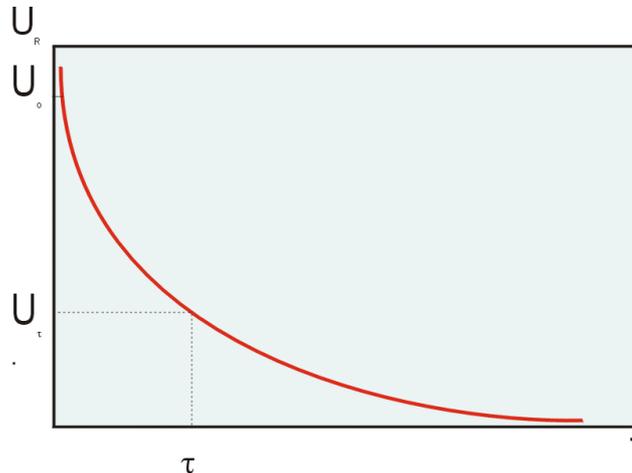Figure 2 shows this exponential decay curve.

Figure 2. Time dependence of the voltage across the resistor U_R.

When time is :

$$t = \tau = RC \quad (2)$$

the initial voltage decreases e times from the initial value:

$$U_\tau = U_0 \exp(-1) \approx 0.37 \cdot U_0 \quad (3)$$

The characteristic time τ is called the time constant of the RC circuit.

The charge delivered during the time t_k to the capacitor comes from the current i :

$$i = \frac{U_R}{R} \quad (4)$$

and is given by:

$$Q = \int_{t_0}^{t_f} i\,dt \quad and \quad Q = \frac{1}{R}\int_{t_0}^{t_f} U_R\,dt \quad (5)$$

So, by examining the $U_R(t)$ dependence one can establish the basic parameters of the RC circuit:
      time constant
      unknown term R or C
      total charge of the capacitor

APPARATUS FOR THE EXPERIMENTAL EXPLORATION.

Exploration of the transients in the RC circuit can be performed in the simple setup consisting of (see Fig.3):

    i. 9V 6LR61 battery
    ii. Capacitor 0.47µF, 16V
    iii. Resistor 40-60 kΩ, 2W
    iv. Calculator Based Laboratory unit CBL
      http://www.vernier.com/legacy/cbl/index.html
      or CBL2
      http://education.ti.com/us/product/tech/datacollection/features/cbl2.html
    v. Voltage probe (standard CBL) http://www.vernier.com/probes/tl.html
    vi. Graphic calculator TI83, TI83 Plus, TI 83 Plus SE.
    vii. unit-to unit cable (standard).
    viii. Lead (single, 10 cm long) for shortening the capacitor.
    ix. Programs: ARC, CALIBRC– available for download at:
    x. TI-GRAPH LINK ^{TM} (optional) cable
      http://education.ti.com/us/product/accessory/connectivity/features/cables.html#serialwin
      and software
      http://education.ti.com/us/product/accessory/connectivity/down/downgraph.html
    xi. Personal computer with TI Connect^{TM} software (optional)
      Description:
      http://education.ti.com/us/product/accessory/connectivity/features/software.html
      Download:
      http://education.ti.com/us/product/accessory/connectivity/down/download.html

Practical notes about the apparatus setup.

- The Voltage probe should be connected to the channel CH1 of the CBL unit.
- Voltage probe connectors should be attached according to the polarity of the voltage.
- The accidental closing of the circuit should be avoided (one may use the independent switch instead of making circuit by hand)
- In case of closing the circuit by hand, do it firmly and keep the circuit closed for approx. 1 second only.
- The capacitor should be discharged (by shortening it with the help of additional lead for not more than 1 sec) each time, before you want to make a measurement.
- One can built more robust permanent circuit at a very low cost instead of set presented on the figure 3.
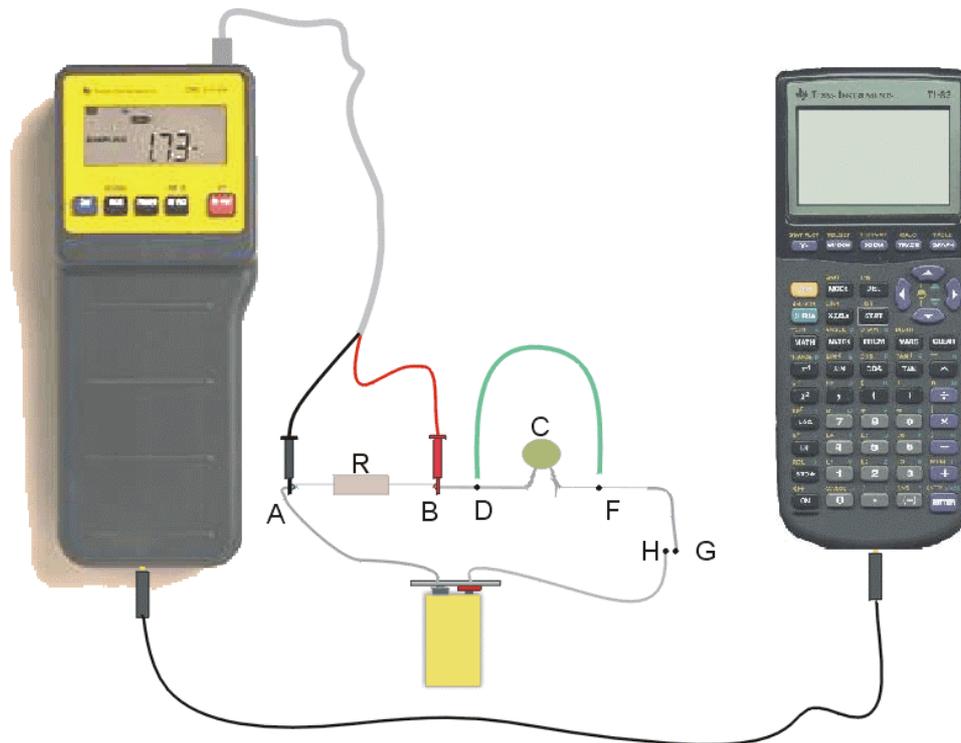
Fig.3. Experimental setup

DATA ACQUISITION (TI 83)

In the experiment the voltage across the resistor is recorded in time after closing the circuit (that is during the charging the capacitor).

The experiment is controlled by the calculator by means of the preloaded calculator programs ARC and CALIBRC.

Experimental procedure is divided into the preparatory part and the data acquisition.

Preparation:

Make all connections as shown on the Fig.3.
Turn on the calculator and the CBL.

Main measurement should be preceded by the two point calibration of the voltage probe.

1. Launch the program ARC by choosing its name from the **PRGM** menu.
2. Choose CALIBRATION form the program menu – Fig.4.
3. Introduce the number of channel CH1 to which the probe is attached:**1** – Fig.5.
4. Choose option **1:PERFORM NEW** – Fig.6.
5. Follow the steps shown on the display (Fig.7.) and introduce first value: **0.0**.
6. Disconnect of the probe connectors from the circuit and perform the second step of the calibration (Fig.8.) Introduce the value **2.0**.
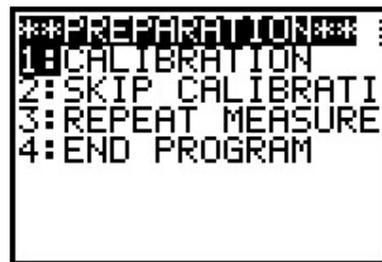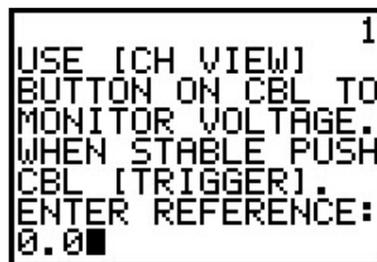
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```
**PREPARATION**
1:CALIBRATION
2:SKIP CALIBRATI
3:REPEAT MEASURE
4:END PROGRAM
```

```
USE LOWEST
AVAILABLE
CHANNELS.

ENTER CHANNEL
NUMBER:1█
```

Fig.4.   Fig.5.

```
**CALIBRATION**
1:PERFORM NEW
2:MANUAL ENTRY
```

```
                    1
USE [CH VIEW]
BUTTON ON CBL TO
MONITOR VOLTAGE.
WHEN STABLE PUSH
CBL [TRIGGER].
ENTER REFERENCE:
0.0█
```

```
                    2
USE [CH VIEW]
BUTTON ON CBL TO
MONITOR VOLTAGE.
WHEN STABLE PUSH
CBL [TRIGGER].
ENTER REFERENCE:
2.0█
```

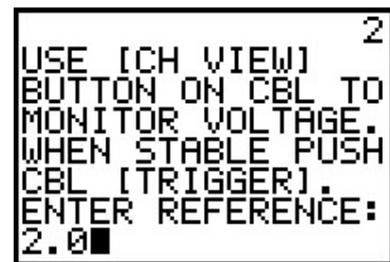Fig.6   Fig.7   Fig.8

The described calibration is to be done once during the experimental session.

As a result of the above procedure the calibration equation is sent to the CBL and will be valid as EQ1 for the voltage probe until next calibration.

Data collection

1. Connect voltage probe to the point **A** and **B** of the circuit (Fig.3.).
2. Choose the sampling time adequate to cover, with the 100 samples to be taken, the time range of charging the capacitor. With the assumed R<100 kΩ and C = 0,47µF the proper sampling time is approx. 0.001sec.
3. Introduce the chosen time interval (Fig.9).
4. Discharge the capacitor by means of lead connected for a while (approx 1sec) to the terminals of the capacitor (see Fig.3. – green lead between the points **D** and **F**).
5. Close a circuit by making a short (0.5 sec) connection between points **G** and **H** (see Fig.3).
6. Trigger sending the collected data to the calculator (fig.10) and plotting a graph U(t) – Fig.11. Time values are stored in the list L_1 and the voltage in the list L_2.
7. Further analysis can be performed after leaving the ARC program by **[ENTER][CLEAR][CLEAR]**
8. If you are not satisfied with he obtained data (A good curve should have no straggled bits), the experiment can be easily repeated using the same or new settings (new calibration is not necessary).
9. Now you can disconnect the CBL from the calculator.
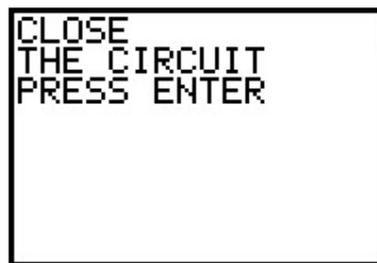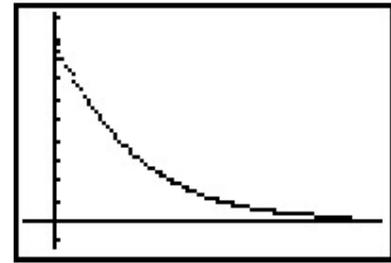
Fig.9   Fig.10   Fig.11

DATA ANALYSIS (TI83)

Further analysis can be performed using tools implemented in calculators (or other analytical software tools such as MS Excel spreadsheet).

Collected data are stored in the calculator's lists:
- time in seconds - List L_1
- voltage U_R in Volts - List L_2

Exemplary data are available for a download in the following calculator type files:

- time : as the file - Data sample/TI83/ **L_1**
- voltage - Data sample/TI83/ **L_2**

Experimental data plot is defined as Plot1 (L_2 vs. L_1) and can be recalled by **[GRAPH]** key. Comparison with the theoretical model (compare Fig.2. and Fig.11) shows that experimental data fit well the model.

Data selection
Collected data contain the pre-stored points which show the voltage over the resistor before the circuit is connected. So, the data set and graph should be tidied up to remove the short horizontal line at the beginning.

Here is a list of instructions:
- Press **[2nd][STAT]** and choose **OPS** and **8: Select** then enter lists' names as parameters:**[2nd] [1][,][2nd][2][)][ENTER]**.
- You can now choose the left bound of the part of the graph you want. Move the marker to the top of the curve using the left and right arrow keys**.** Press **[ENTER].**
- Choose the right bound by moving the marker as far to the right as possible and press **[ENTER]** again.
- Press **[ZOOM]**, choose **9:ZoomStat** and press **[ENTER]** - Fig.12.
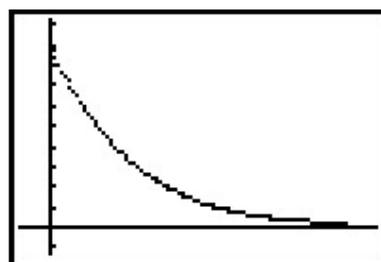

Fig.12.

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Experimental module: **Transients in the RC circuit**

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Analysis of plots

Transients in the RC circuit are described by the characteristic time parameter called Time constant

A. Establishing the time constant τ directly from the experimental curve on the basis of the eq. 3

- Turn on the plot **[GRAPH]**. Pick up a start voltage, U_0, using **[TRACE]** key (Y-value). Note the time, (X-value).
- Work out $e^{-1} \cdot U_0 = (0.37 \cdot U_0)$.
- Find this voltage on your graph using **[TRACE]** key.
- Use the graph to work out how long it took for the voltage to fall to this value, pick up the corresponding time value. This is the time constant of the circuit, $\tau = R \cdot C$.
- Record this value as τ_{graph}

B. Establishing the time constant τ by the numerical analysis of the experimental data

From the eq.1 comes that the time constant $\tau = R \cdot C$ can be calculated as a slope of the linearised $U_R(t)$ dependence – $ln(U_R) = f(t)$.

$$\ln U_R = \ln U_0 + (-\frac{1}{RC})t \quad (6)$$

which follows the general linear form :

$$y = ax + b \quad (7)$$

where: $b = \ln U_0 \qquad and \qquad a = -\frac{1}{RC} \quad (8)$

With the features of the graphic calculator you can perform this analysis easily by the linear regression function.

1. Linearisation of the U(t) dependence calls for calculation of the ln of each U_R value from the list L_2. The calculated values should be stored in a separate list e.g. L_3.
   Perform:**[LN][2nd][2][)][STO->][2nd][3]** (Fig.13).
2. To display the $ln(U)$ vs time one should define the new active plot Plot2 as L_3 vs. L_1 (Fig.16). Note: Plot1 should be put into inactive mode.
3. Linear regression y=ax+b for data from lists L_1 and L_3 calls for sequence:
   [STAT][4]
   [2nd] [1][,][2nd][3][,][VARS][>][1][1][ENTER] (Fig.14).
4. The equation for the linear dependence is shown (Fig.15). The meaning of the displayed coefficients *a* and *b* comes from the eq. 8 while the *r* is the correlation coefficient of the linear regression. The established equation is stored as the Y_1 function variable.
5. Recalling the graph one can see data points and regression line on one plot (Fig.17).
6. From the *a* value you can establish the time constant of the circuit τ_{reg} (see eq.2 and 8). Compare the obtained time constant with its value obtained directly from the examining the graph τ_{graph}.
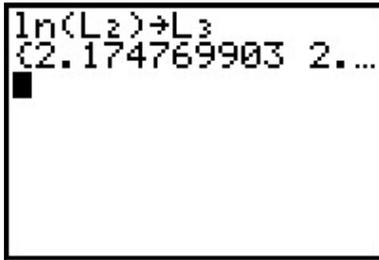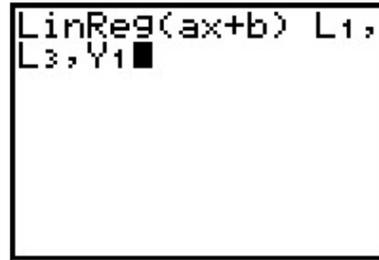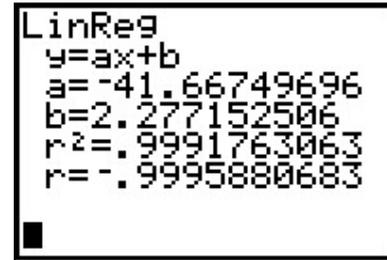
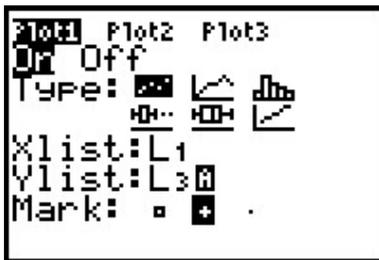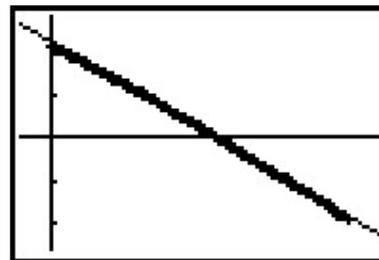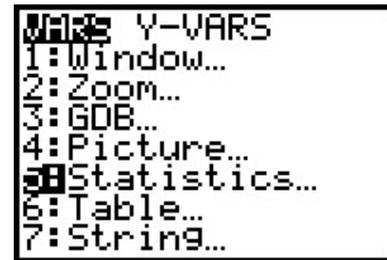Fig.13

Fig.14

Fig.15

Fig.16

Fig.17

Fig.18

C. Calculation of the resistance R from the time constant and known capacitance C.

Now if you know C ($0{,}47\mu F$), you can work out R from the time constant value. Notice that the time constant is independent of U_0.
Note:
The established slope *a* of the regression can be recalled from the menu **[VARS] 5:Statistics** (Fig.18).

D. Calculation of the charge gathered by the capacitor

Charge gathered by the capacitor in the observed time can be established on the basis of the eq. 5 with the help of the definite integral function performed by the calculator. This requires the explicit form of the integrand function $U_R(t)$. The general form is given by eq. 1. The required terms: U_0 and $(-1/RC)$ were established in the step B (see eq. 8.). These values once calculated from the regression procedure are stored in the calculator's memory and could be used in constructing the function $U_R(t)$:

$$U_R = e^b \cdot e^{at} \quad (9)$$

This function should be defined and stored in the memory of the device.

Here is a list explaining what to do:
1. Start the equation editor: Press **[Y=]**, choose the position Y $_2$.
2. Introduce function *exp* by: **[2nd][LN]**, then its exponent *b* by:**[VARS] 5:Statistics [ENTER]** **[ENTER]** choose **EQ** and **b [ENTER] [)]**
3. Repeat introducing *exp* by **[2nd][LN]**, then exponent *at* by: **[VARS] 5:Statistics [ENTER EQ** and *a* **[ENTER][X,T,θ,n] [ENTER][)]**.
4. Activate the function Y$_2$: place the cursor at the **=** symbol and press **[ENTER]** (in a active mode the **=** symbol is blackened).
5. Deactivate the function **Y$_1$** : place the cursor at the **=** symbol and press **[ENTER]** (in a no active mode the **=** symbol is displayed in a normal way).
6. Function U$_R$(t) is now defined and stored as Y$_2$ (Fig.19).

Now one can see the plot of this function together with the original experimental data U(t). Follow the instruction below.

1. Change the active plot into PLOT1 (with Xlist = L_1 , Ylist = L_2).
2. Make a graph:**[ZOOM][9]**.
   You should now see both the experimental data and the plot of the $U_R(t)$ function (Fig.20).

Calculation the definite integral. (eq.5)

Charge gathered on the capacitor can be calculated with the help of the integral of the U(t) function within the chosen range of time.
Here is a list explaining what to do.
1. Choose integration from the **CALC** menu:**[2nd][TRACE] 7:∫f(x)dx [ENTER]** (Fig.21). Plot is now displayed.
2. Choose the lower limit of integration. Move the cursor marker to the top of the curve using the left and right arrow keys. Press **[ENTER]**. Record this time value: t_0.(X-value).
3. Choose the upper limit of integration by moving the cursor as far to the right as possible and press **[ENTER]**. Record this time value: t_k (X-value).
   The area below the curve becomes dark and the definite integral value is now displayed (Fig.22).
4. On the home screen recall the established value of the integral: **[2nd][-][ENTER]**.
5. According to the eq.5 from this value you can calculate the value of the charge
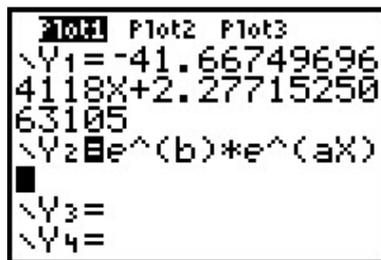   delivered to the capacitor in the chosen time (defined by limits of integration): Q=Integral /R.
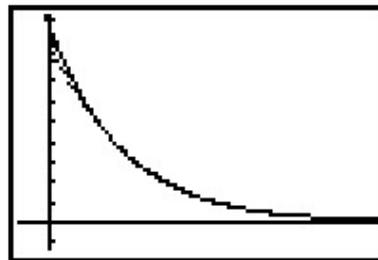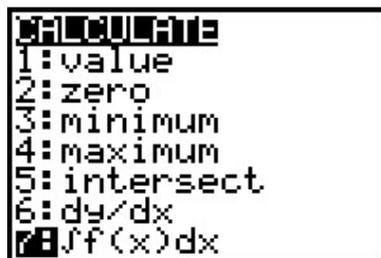

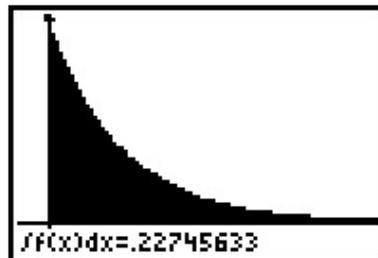
Fig.19

Fig.20

Fig.21

Fig.22

DATA ANALYSIS (MS EXCEL)

Further analysis can be performed using tools implemented in calculators and personal computer functions offered by the MS Excel software.

Transfer of experimental data to the personal computer.

After completing the experiment data could be transferred from the graphic calculator to the personal computer.

TI GRAPH LINK^{TM} cable supported by the TI Connect^{TM} software offer tools enabling exploration of the contents of calculator (TI DEVICE EXPLORER) and data edition (TI DATA EDITOR).

Data collected in the experiment are stored in the calculator's lists:
- time in seconds - List L_1
- voltage in Volts - List L_2

Within the TI Connect^{TM} program - the TI DEVICE EXPLORER one can save calculator's lists on the computer's hard disk and then open them within TI DATA EDITOR. Option **Special Lists Export** provides opportunity to save the chosen list as the Excel Comma Separated type file (*CSV file). Such a file could be open and explore within the MS Excel^{TM} spreadsheet software.

Exemplary data are available for a download in the following files:

- time : as file - Data sample/MSExcel /**time**
- voltage: as file - Data sample/MSExcel/**voltage**

Analysis of plots

Transients in the RC circuit are described by the characteristic time parameter called Time constant

E. Establishing the time constant τ directly from the experimental data on the basis of the eq. 3

- Create a spreadsheet and import the data from the files **time** and **voltage.** Make the plot U(t) from the experimental data – Fig. 23. Pick up a start voltage, U_0, record the time, (X-value).
- Work out $e^{-1} \cdot U_0 = (0.37 \cdot U_0)$ -eq. 3.
- Find this voltage on your voltage column.
- From the time column, pick up the corresponding time value. This is the time constant of the circuit, τ = R·C.
- Record this value as τ_{graph}

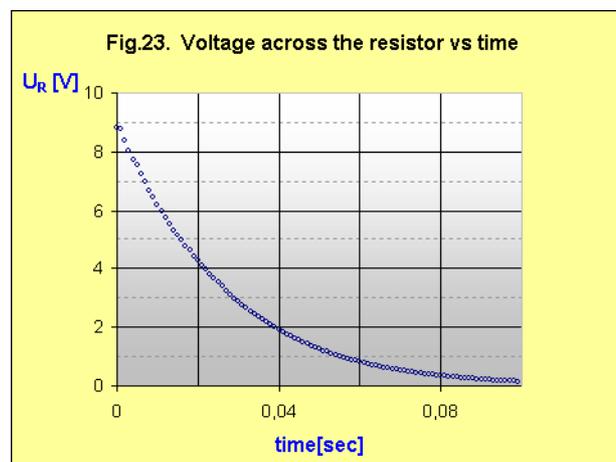

Fig.23.

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F. Establishing the time constant τ by the numerical analysis of the experimental data

From the eq.1 comes that the time constant τ = R·C can be calculated as a slope of the linearised $U_R(t)$ dependence – $ln(U_R)=f(t)$ – eq. 6,7,8.

With the feature of the MS Excel you can perform this analysis easily by means of the linear regression.

1. First the *ln* of each U_R value should be calculated and kept in the separate column.
2. The new scattered plot of *ln(U)* vs time should be defined – Fig.24.
3. Ask for display of the trend line. Choose linear regression. The equation of the linear approximation is shown in the plot area -Fig.24.
4. Now you can compare data points and regression line on one plot (Fig.24).
5. From the *a* value you can establish the time constant of the circuit τ_{reg} (see eq.2 and 8). Compare the obtained time constant with its value obtained directly from the examining the graph τ_{graph}.
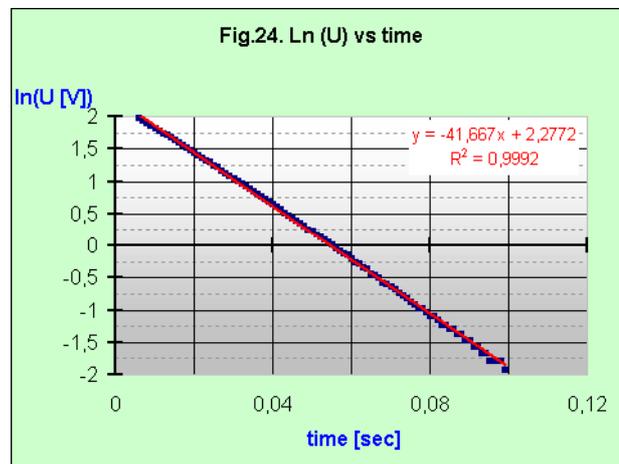

Fig.24

G. Calculation of the resistance R from the time constant and known capacitance C.

Now if you know C (0,47μF), you can work out R from the time constant value. Notice that the time constant is independent of U_0 – eq.8.

H. Calculation of the charge gathered by the capacitor

First we recommend to calculate the values of the voltage using the explicit form of the $U_R(t)$. The general form is given by eq.1. The required terms: U_0 and *(-1/RC)* were established in the step F (see eq. 8.). The new column should be defined containing the values $U_R(t)$ as function of the cells containing time values - eq.9.

Now one can make another plot with two series: original experimental data points and function U(t) – Fig.25. Comparison of the analytical dependence and experimental data shows good agreement with the exponential decay model.
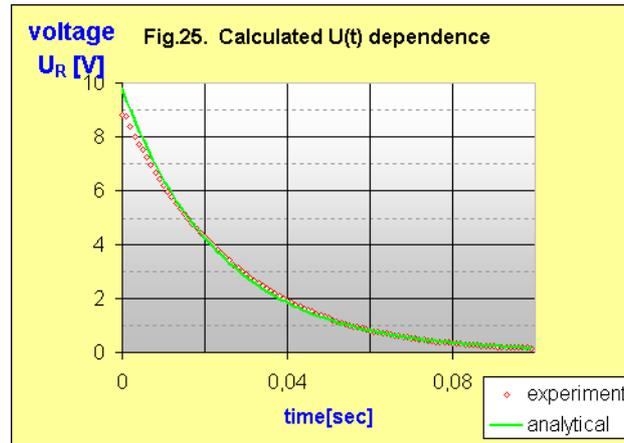
Fig.25.

The charge gathered by the capacitor in the observed time can be established with use of the eq.5.

Definite integral of the U(t) function values can be calculated using the trapezoid rule – eq.10.

$$Q \cdot R = \int_{t_0}^{t_f} U_R dt = \sum_{i=1}^{n-1} \frac{(U_i + U_{i+1})}{2} \cdot \Delta t \qquad (10)$$

The integral values should be calculated in the separate column using the eq. 10. The last (bottom) cell contains the calculated integral.

According to the eq.5 from this value you can calculate the value of the charge delivered to the capacitor in the chosen time: Q=Integral/R.

Note:

The complete numerical analysis is presented in the MSExcel file: Data sample/MSExcel /**rcanalysis**