

Newton's law of cooling



Experimental setup and procedure

Objective

The objective of this experiment is to investigate the change in temperature when an object cools and to find a mathematical model to fit the change.

Materials

Thermo flask with boiling water, temperature-probe, CBL and TI-83. As an alternative to boiling water you can use a hair-drier.

Procedure

- Before setting up the experiment make sure that you have the programs [TEMP](#) and [CLEAN](#) in your TI-83. If not download them. Short instruction on how to download [here](#).
- Connect the temperature-probe to channel 1 on the CBL and connect the CBL to the calculator.
- Put the temperature-probe in the boiling water without touching the heater or heat it with the hairdryer.
- Start the program TEMP on the calculator.
- Choose the scale to AUTO and choose a suitable number of measurements per second, for example 1.
- Take the probe out of the water, move it quickly out of the steam, dry it quickly and push ENTER, alternatively, move it away from the hot air from the hair-drier and push ENTER.
- When the experiment is done the temperature ($^{\circ}\text{C}$) is shown as a function of time (s). Temperature data are stored in list L_2 and the time data are stored in list L_1 .
- Do not forget to register the room temperature.

If you are unable to conduct the experiment there are files set up so that you can look at the experiment and analyse the data from it. Choose from the menu below

- Get data to your TI83 or TI83 Plus. [Short instruction how to download](#). You will find the data you need in folder Newton law of cooling.

Analysis

To go to the analysis section choose one of the options below:

- [Data analysis using TI83/TI84](#) (your own or downloaded data)
 - [Data analysis using Excel](#)
-

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Data analysis (TI 83/TI84)

If you have any problems handling the graphing calculator help is available using the links that are underlined and highlighted in blue.

- Plot the temperature as a function of time in a [scatter plot](#).
- Look at the graph. Does it meet your expectations? Do cooling rates at different temperatures look reasonable? How far down will the temperature go?

- To be able to fit a function to the values the room temperature has to be subtracted from all temperature data. We suggest that you put the new subtracted values in a new list, not destroying the experimental data. To do this position the cursor at the top of list L₃ and type L₃ = L₂ - "room temp".

L1	L2	L3
0	80.69	-----
1	80.24	
2	79.26	
3	78.58	
4	77.7	
5	76.84	
6	75.94	

L3 = L2 - 19

- Now graph the temperature difference between the probe and room temperature as a function of time in a [Scatter plot](#).
- Perform a suitable [regression](#) to the measured values and try to make a conclusion.
- To calculate the cooling rate in each data point we can use the [dy/dx-function](#). This is found in CALC (2:nd TRACE) and give the slope of the function. After using it the result is stored in the answer register and is accessed by pressing ANS (2:nd (-)). An easy way to put these data into new lists is to:
 - perform the dx/dy on a point
 - go to the list editor (STAT Edit)
 - position cursor at L₆(1) and press ANS (2:nd (-)). This pastes the slope into the cell.
 - position cursor in L₅(1) and press Y (ALPHA 1). This pastes the temperature difference into the cell
- Make a [scatter plot](#) showing the cooling rate as a function of the temperature difference to room temperature.
- Perform a suitable [regression](#) to the measured values and try to make a conclusion.

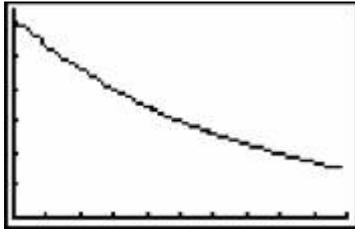
When you have finished your analysis you can compare it with this [completed analysis](#)

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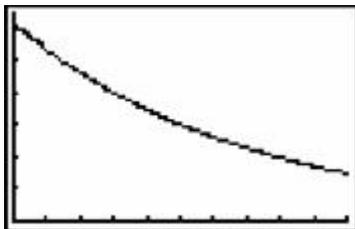
Completed analysis - (TI 83/TI84)

After having subtracted the room temperature from the collected data our scatter plot looks like this:



Not surprising we find a higher cooling rate when the difference in temperature to the environment is higher. We also see that it is reasonable to suppose that there is an exponential dependence.

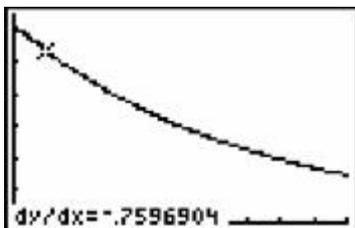
We then fit an exponential curve to the data using [exponential regression \(ExpReg\)](#). Doing that gives us the following:



```
ExpReg
y=a*b^x
a=61.53128895
b=.985865652
```

As seen in the left picture our regression curve fits the data almost perfectly. The function is $y = 61.5 \cdot 0.986^x$, where y is the temperature difference to room temperature and x is the time.

To look at the cooling rate we use the [dy/dx-function](#) on the calculator. This allows us to take a numerical derivative in any point of the graph.

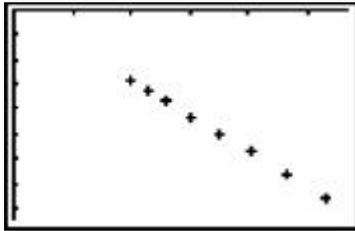


L4	L5	L6	5
53.367	-.7597	-----	
<hr/>			
L5(2) =Ans			

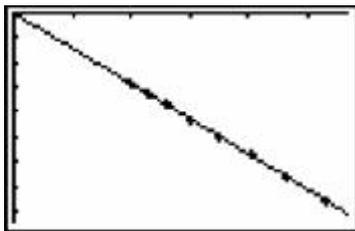
The dy/dx performed in the point where $x = 10$

The dy/dx value is pasted into a list using ANS (2:nd (-)). At the same time the y -value is pasted in another list using Y (2:nd 1)

After doing this with a number of points we [graph the cooling rate \(the derivative\) as a function of the temperature difference](#).



Looking at this we might get the idea that it is a linear function and we therefore try a [linear regression](#) on the data. That will give us:



```
LinReg
y=ax+b
a=-.0142351893
b=2.7716E-10
```

The equation of the linear function is $y = -0.0142 \cdot x$ which means that the cooling rate is $-0.0142 \text{ } ^\circ\text{C/s}$ per difference in degrees to the environment, i.e., -0.0142 s^{-1} .

The cooling rate can be expressed as a derivative $\frac{dT_c}{dt}$

Therefore we can write the equation as: $\frac{dT_c}{dt} = -k \cdot (T_c - T_0)$ with $k = 0.0142 \text{ s}^{-1}$.
 T_c is the temperature in $^\circ\text{C}$ and T_0 the room temperature.

The cooling rate is proportional to the difference in temperature between the object and the environment. This equation is the *Newton's law of cooling*.
