

EXPLORING FARADAY'S LAW

OBJECTIVE

Experimental studies of the Faraday's law of induction. The Faraday's law of induction is examined by observations of the electromotive force (emf) induced in the coil by the oscillating bar magnet. The time dependence of the magnetic flux density due to the moving magnet and time dependence of the emf. induced in the coil are studied. Exploration of the emf dependence on the time rate of change of the magnetic field flux penetrating the coil is used for examining the Faraday's law.

CONTENTS

- I. Theoretical model.
- II. Apparatus for the experimental exploration.
- III. Data acquisition.
- IV. Data analysis (TI83)
- V. Data analysis (MS Excel)

THEORETICAL MODEL

Faraday's law states that the electromotive force induced in the closed circuit affected by the magnetic field is directly proportional to the time rate of change of the magnetic field flux penetrating the circuit.

The emf induced in the circuit is equal to the negative time derivative of the magnetic field flux through the circuit. (Eq. 1.)

$$E = - \frac{d\Phi_B}{dt} \quad (1)$$

The negative sign in the equation determines the direction of the current induced in the circuit. It is ruled by the **Lenz's law** which states that the direction of the induced current is such as that it's magnetic effect opposes the cause that produces the current.

The change of the magnetic field flux could be due to the change of at least one of its factors (Eq.2).

$$\Phi = \vec{B} \circ \vec{S} = B \cdot S \cdot \cos \angle(\vec{B}, \vec{S}) \quad (2)$$

$$\frac{d\Phi}{dt} = \frac{d}{dt} (B \cdot S \cdot \cos \angle(\vec{B}, \vec{S})) \quad (3)$$

Thus, the emf could be induced in variety of cases. Typical examples are: changing magnetic field and relative motion of the magnetic field source and the circuit. The induction effect due to the rotatory motion of the loop in the magnetic field is functional principle of the electrical generators.

The detailed discussion of the emf induced in the present experiment is given by the [Eq.9](#).

Magnetic field of the bar magnet.

Spatial distribution of the magnetic field lines produced by the bar magnet is similar to that one due to the coil with the current - Compare figures 1 and 2. In particular, magnetic flux density \mathbf{B} of the bar magnet can be presented in terms of the field produced by the magnetic dipole characterized by the magnetic dipole moment μ . Magnetic flux density (magnetic induction) along the axis of the magnet at the distance y from it in vacuum is given by:

$$\vec{B} = \frac{\mu_0}{2\pi y^3} \mu = C \frac{1}{y^3} \quad (4)$$

Magnetic induction on the axis of the coil consisting of N circular loops of radius r with the current i is given by the similar equation:

$$B_y = \frac{\mu_0 N i r^2}{2(y^2 + r^2)^{3/2}} \quad (5)$$

Defining magnetic dipole moment of the coil as:

$$|\mu| = N i \pi r^2 \quad (6)$$

the expression for the magnetic induction, for the points far away from the center of the coil, where $y \gg r$, becomes:

$$\vec{B}_y = \frac{\mu_0 \mu}{2\pi y^3} \quad (7)$$

Which is essentially the same result as for the bar magnet (Eq. 4)

Magnetic field due to the oscillating bar magnet.

Magnetic induction at the certain point in space due to the bar magnet moving with respect to that point is changing in time following the change in time of the distance from the point concerned to the center of the magnet.

Fig 3. presents situation in the presented experimental observations.

Assuming that:

D - is the distance from the coil to the equilibrium position of the magnet.

A - is the amplitude of the magnet's oscillations

ω - is the angular frequency of the oscillations

φ - is the initial phase of the oscillations

y - the distance to the coil from the present position of the magnet is given by:

$$y = D + A \sin(\omega t + \varphi) \quad (8)$$

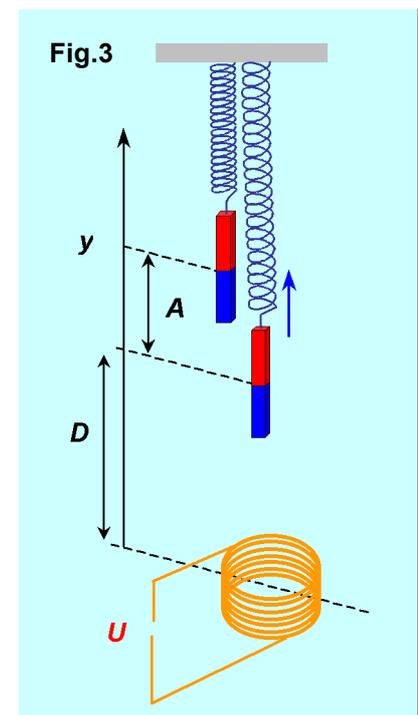
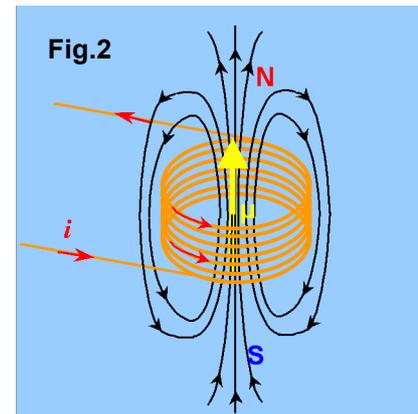
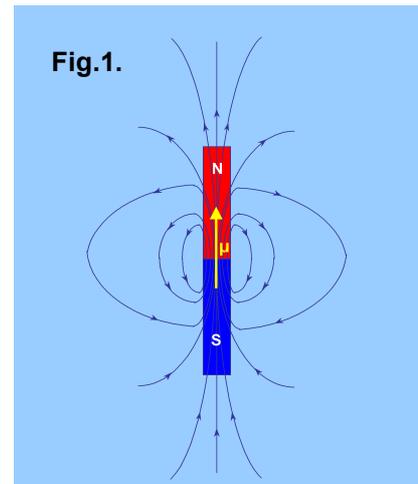


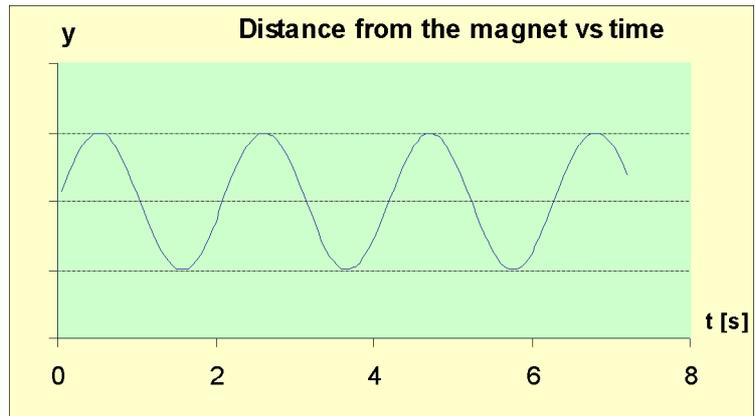
Figure 4 shows distance vs. time dependence

The magnetic field in the center of the coil is given by (compare Eq. 4):

$$B_y(t) = C \frac{1}{(D + A \sin(\omega t + \varphi))^3} \quad (9)$$

Figure 5 shows induction vs. time dependence

Fig.4.

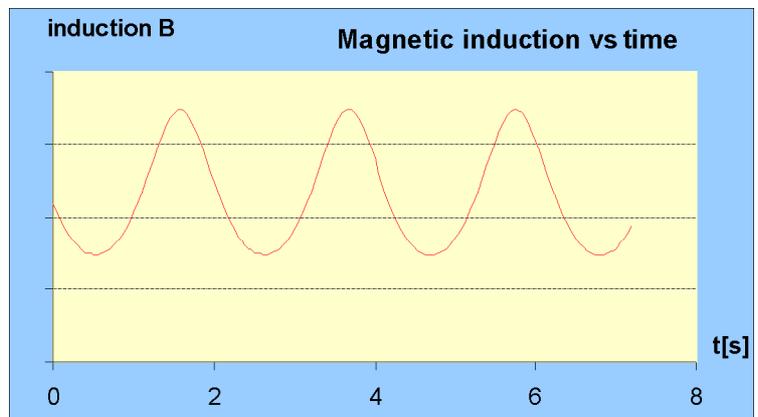


A time rate of the magnetic field induction at the position of the coil due to the oscillating magnet can be obtained analytically from Eq.9 as dB/dt to be:

$$\frac{dB}{dt} = C \frac{-3Aw \cos(\omega t + \varphi)}{(D + A \sin(\omega t + \varphi))^4} \quad (10)$$

Figure 6 shows dB/dt vs. time dependence.

Fig.5.



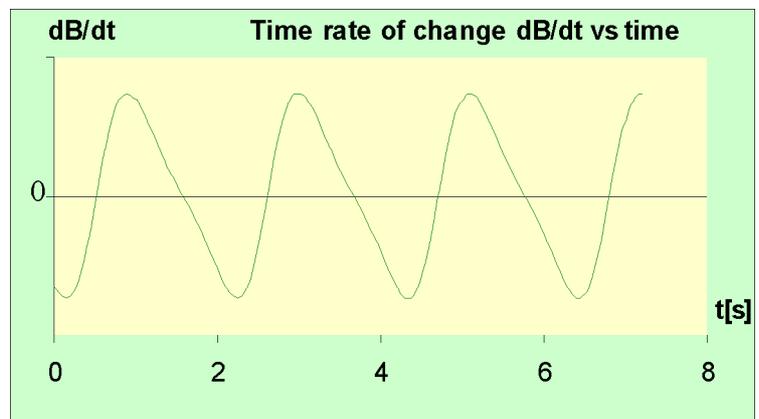
Assuming that orientation and geometrical parameters of the coil do not change in time the magnetic field flux through the coil in this situation is described by equation:

$$\Phi = NS \cdot C \frac{1}{(D + A \sin(\omega t + \varphi))^3} \quad (11)$$

where:

N - is the number of turns
S - is the cross section area of the single turn of the coil.

Fig.6.



Note:

Equation 11 is based on the simplifying assumption that the magnetic field is the same for all coil's turns.

Thus the time rate of change of the magnetic field flux through the coil is given by:

$$\frac{d\Phi}{dt} = NS \cdot C \frac{-3Aw \cos(\omega t + \varphi)}{(D + A \sin(\omega t + \varphi))^4} \quad (12)$$

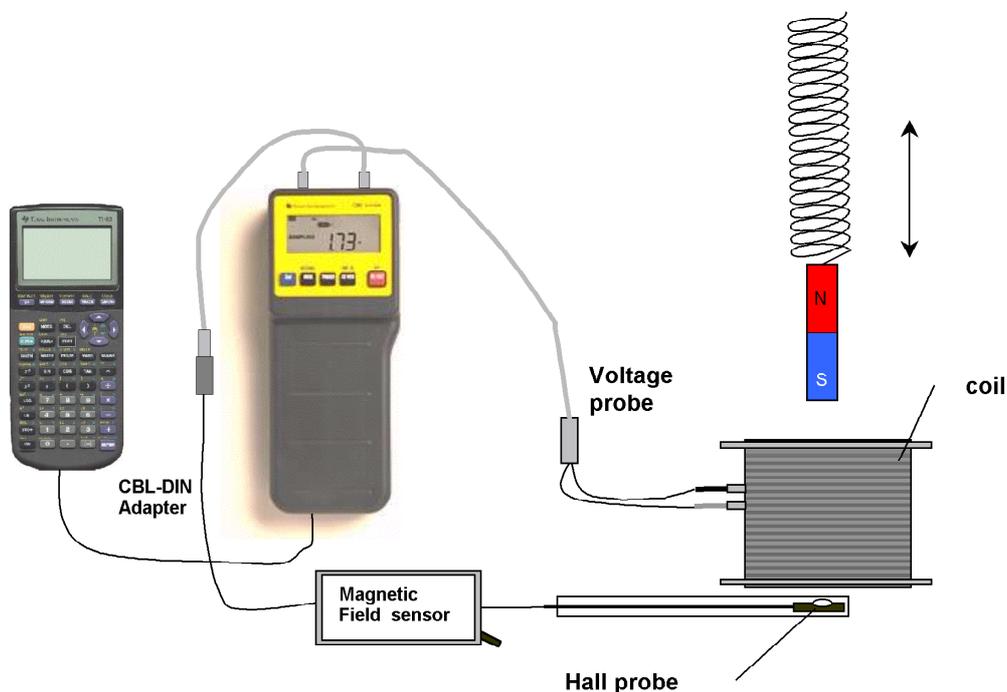
and the plot $\frac{d\Phi}{dt}$ vs. time is similar to that presented on the Figure 6.

APPARATUS FOR THE EXPERIMENTAL EXPLORATION.

Exploration of the electromagnetic induction phenomenon can be performed in the simple setup consisting of (see Fig.7):

- i. Coil, e.g. 6500 turns ($d=0.2\text{mm}$), cross sectional area 20 cm^2 , diameter 5cm
- ii. Laboratory bar magnet (10 cm long)
- iii. Spring (30 cm long - equilibrium) making with the magnet an oscillator with period about 1-3 sec
- iv. Laboratory stand (approx. 1m high)
- v. Calculator Based Laboratory unit CBL
<http://www.vernier.com/legacy/cbl/index.html>
or CBL2
<http://education.ti.com/us/product/tech/datacollection/features/cbl2.html>
- vi. Magnetic Field Sensor for CBL™ (Vernier MG-DIN or MG-BTA)
<http://www.vernier.com/probes/mg.html>
- vii. CBL - DIN Adapter (for MG-DIN only)
<http://www.vernier.com/probes/adapters.html>
- viii. Voltage probe (standard CBL) <http://www.vernier.com/probes/ti.html>
- ix. Graphic calculator TI83, TI83 Plus, TI 83 Plus SE, TI89, TI92, TI92 Plus, TI Voyage 200
- x. unit-to unit cable (standard)
- xi. program Physics (version relevant to the calculator)
<ftp://ftp.ti.com/pub/graph-ti/cbl/programs/83/>
<ftp://ftp.ti.com/pub/graph-ti/cbl/programs/83plus/>
<ftp://ftp.ti.com/pub/graph-ti/cbl/programs/89/>
<ftp://ftp.ti.com/pub/graph-ti/cbl/programs/92/>
<ftp://ftp.ti.com/pub/graph-ti/cbl/programs/92plus/>
- xii. TI-GRAPH LINK™ (optional) cable
<http://education.ti.com/us/product/accessory/connectivity/features/cables.html#serialwin>
and software
<http://education.ti.com/us/product/accessory/connectivity/down/downgraph.html>
- xiii. Personal computer with TI Connect™ software (optional)
Description:
<http://education.ti.com/us/product/accessory/connectivity/features/software.html>
Download:
<http://education.ti.com/us/product/accessory/connectivity/down/download.html>

Fig.7.



Practical notes about the apparatus setup.

- Bar magnet hanging on the spring is freely oscillating up and down along the axis consistent with the axis of the laboratory coil.
- The South magnetic pole of the magnet should be faced towards the coil.
- The minimal distance from the top of the coil to the magnet during the motion should not be smaller than 3 cm.
- Other oscillation's parameters should be approx.: the amplitude 10 - 15 cm, period 1-3 sec.
- The Magnetic field probe should be firmly mounted (e.g. with the tape) below the center of the coil with its white mark facing upwards (towards the South magnetic pole of the magnet).
- The switch on the Magnetic field sensor box should be put into the "Low" mode.
- The Magnetic Field Sensor should be connected via the CBL-DIN adapter to the Channel CH1 of the CBL unit.
- The Voltage probe should be connected to the Channel CH2 of the CBL unit.

DATA ACQUISITION (TI 83)

In the experiment the induction of the magnetic field produced by the bar magnet is recorded together with the voltage induced in the coil due to the change of the penetrating magnetic field flux. Both the magnetic induction and the voltage are recorded simultaneously as functions of time. The experiment is controlled by the calculator by means of the preloaded calculator program Physics.

Experimental procedure is divided into the preparatory part and the data acquisition.

Preparation:

The program Physics should be launched either by choosing its name from the PRGM menu (TI83) or from Application menu: APPS (TI83 Plus).

From the main menu one should choose the 1: SET UP PROBES (Fig.8.) option and number of probes in option 2: TWO (Fig. 9) . The options - 2; MAGNETIC FIELD probe and then 3: Voltage probe should be chosen (Fig.10).

```

***MAIN MENU***
1:SET UP PROBES
2:COLLECT DATA
3:ANALYZE
4:TRIGGERING
5:ZERO PROBES
6:RETRIEVE DATA
7:QUIT
    
```

Fig.8

```

NUMBER OF PROBES
1:ONE
2:TWO
3:THREE
    
```

Fig.9

```

SELECT PROBE
1:LIGHT
2:MAGNETIC FIELD
3:VOLTAGE
4:C-V CURRENT
5:C-V VOLTAGE
6:THERMOCOUPLE
7:MORE
    
```

Fig.10

Main measurements must be preceded by the calibration of the probes.

In the present experiment, which presumes qualitative explorations only, the prestored standard calibration settings can be used. Choose then 1: USE STORED (Fig. 11.) and 4: LOW (MTESLA) (Fig.12) settings.

Both probes should be set to zero in the absence of the bar magnet. This avoids the effect of the Earth magnetic field magnetic field on the readings. So, move the bar magnet far away from the coil and the probe and chose option 5: ZERO PROBES (Fig. 13.) and then select 5: ALL CHANNELS (Fig.14). When the values displayed at the CBL become constant press black button TRI GGER below the CBL display.

```

***CALIBRATION***
1:USE STORED
2:PERFORM NEW
3:MANUAL ENTRY
    
```

Fig.11

```

MG FIELD SETTING
1:HIGH(GAUSS)
2:HIGH(MTESLA)
3:LOW(GAUSS)
4:LOW(MTESLA)
    
```

Fig.12

```

***MAIN MENU***
1:SET UP PROBES
2:COLLECT DATA
3:ANALYZE
4:TRIGGERING
5:ZERO PROBES
6:RETRIEVE DATA
7:QUIT
    
```

Fig.13

The described calibration is to be done once during the experimental session.

Setting up the data collection parameters:

From the program main menu choose option 2; COLLECT DATA (Fig.13) and 2: TIME GRAPH (Fig.15) in order to record channels readings vs. time. Input sampling time (e.g. 0.04sec) and number of samples (e.g. 50) (Fig. 16).

Before triggering the data collection (Fig. 17) you should check adjustments of the apparatus and make the bar magnet performing the oscillations (up and down).



Fig.14



Fig.15

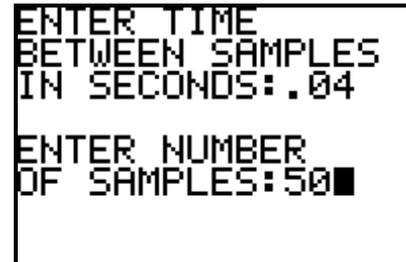


Fig.16

Measurements are completed automatically and the relevant information is seen on the calculator's screen (Fig. 18).

Time values are stored in the list L_1 , magnetic induction in the list L_2 and the voltage induced in the coil as L_3 .



Fig.17



Fig.18.



Fig.19.

You can produce the graph of the magnetic induction vs. time by choosing from the menu (Fig. 19) 1: CHANNEL 1 - (Fig. 20) or graph of the voltage (induced emf) vs. time by choosing 2: CHANNEL 2 (Fig. 21).

If you are not satisfied with the obtained data, the experiment can be easily repeated using the same or new settings (new calibration is not necessary) (Fig. 22).

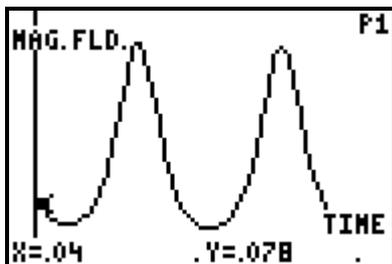


Fig.20

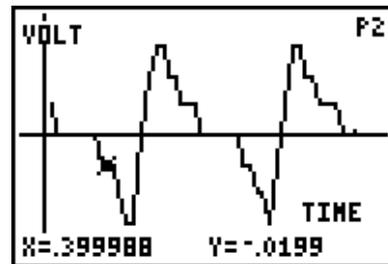


Fig.21



Fig.22

Further analysis can be performed after leaving the Physics program.

DATA ANALYSIS (TI83)

Further analysis can be performed in terms of [Faraday's law](#) of induction using tools implemented in calculators (or other analytical software tools such as MS Excel spreadsheet).

Collected data are stored in the calculator's lists:

- time in seconds - List L_1
- magnetic induction in mTesla - List L_2
- voltage (induced emf) in Volts - List L_3

Exemplary data are available for a download in the following calculator type files:

- time : as the file - Data sample/TI83/ L_1
- magnetic induction: as the file - Data sample/TI83/ L_2
- voltage: as the file - Data sample/TI83/ L_3

Analysis of plots

Measured magnetic induction is proportional to the magnetic field flux penetrating the coil. As the source of the field is moving with respect to the coil the field flux is changing in time. As a result of the motion of the bar magnet the induced electromotive force emf is measured as a voltage in the coil. Time dependence of as well as mutual dependence of the magnetic induction and induced voltage can be explored using graphs.

Magnetic field induction (and magnetic field flux) and induced emf vs. time.

Defining Plot1 as L_2 vs. L_1 will result in $B(t)$ graph while defining Plot2 as L_3 , vs. L_1 will result as $emf(t)$ graph. (Fig.23). One could explore both plots (Fig.24) after adjusting the window.

Magnetic induction presented in the Plot1 shows time pattern, which corresponds to the harmonic motion of the magnet.

Voltage presented in the Plot2 reveals similar time pattern with the 'phase' shift.

The change of the slope of the $B(t)$ curve results in the shape of the $emf(t)$ plot. The observed changes in sign correspond to the character of change of magnetic field and can be discussed in terms of the Lenz's law.

Comparison with the theoretical model (compare [Fig 5](#) and [Fig. 6](#)) shows that experimental data fit well the model.

However only the detailed analysis of the direction of the coils turns and polarity of Voltage probe connections enables checking validity of the [Lenz's law](#).



Fig.23.

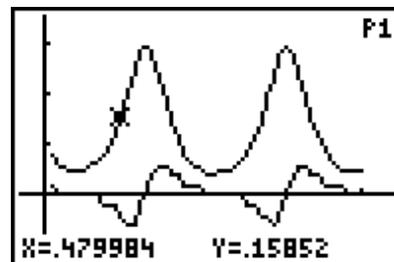


Fig.24.

Checking the Faraday's law

Collected data enable examining **Faraday's law** of induction by checking the proportionality of the time rate of change of the magnetic field flux.

The analysis requires comparing the induced voltage with the time derivative of the magnetic field recorded by the magnetic field probe.

Proposed procedure of numerical differentiation is taken after G. Adie [in 'Using the TI89 in Physics' Bk Techware, 2000]. It is based on calculation of the difference quotients from the original magnetic field data.

The procedure steps are:

1. defining new list L_5 with difference quotients of the subsequent values from the list L_2 calculated for the used time intervals (e.g. 0.04 sec) and
2. defining new lists L_4 and L_6 with the time and voltage data respectively, reduced in size by one (this step follows the preceding reduction of size of the list L_5 imposed by operation in the step 1).

Two special functions $\Delta List()$ and $cumSum()$ used in the described procedure are invoked from the STAT OPS menu – Fig. 25. The command syntax is seen in Fig.26.

As a result three new lists are defined. They contain:

- modified time - list L_4 ,
- time derivative of the magnetic induction -list L_5
- modified induced emf (voltage) - list L_6

Exploration of the Faraday's law calls for a new plot (Plot3) displaying induced emf vs. time derivative of the magnetic induction (L_5 , vs. L_6) (Fig. 27) .

```

NAMES OPS MATH
1:SortA(
2:SortD(
3:dim(
4:Fill(
5:seq(
6:cumSum(
7:ΔList(
    
```

Fig.25.

```

ΔList(L2)/.04→L5
(-.38065 -.1790..
L1(1)+cumSum(ΔLi
st(L1))→L4
(.0799936 .1199..
L3(1)+cumSum(ΔLi
st(L3))→L6
    
```

Fig.26.

```

STAT PLOTS
1:Plot1...Off
  ↗ L1 L2 .
2:Plot2...Off
  ↗ L1 L3 .
3:Plot3...On
  ↗ L5 L6 +
4:PlotsOff
    
```

Fig.27.

From the obtained plot (Fig. 28) one can formulate hypothesis of quasi linear dependence between concerned quantities. This linearity can be examined with the aim of the linear regression function $LinReg(ax+b)$ for lists L_5 and L_6 (from the STAT CALC menu – Fig. 29 and Fig.30).

The derived linear approximation (Fig. 31.) can now be compared with the real data (Fig.32).

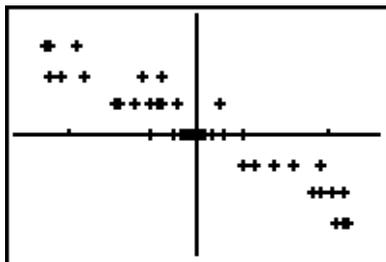


Fig.28.

```

EDIT CALC TESTS
1:1-Var Stats
2:2-Var Stats
3:Med-Med
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7:QuartReg
    
```

Fig.29.

```

LinReg(ax+b) L5,
L6, Y1
    
```

Fig.30.

Slope of the line represents parameters of the coil and can be compared with the theoretical model- see eq.12.

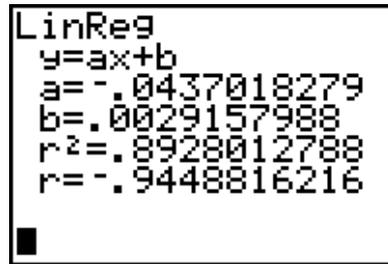


Fig.31.

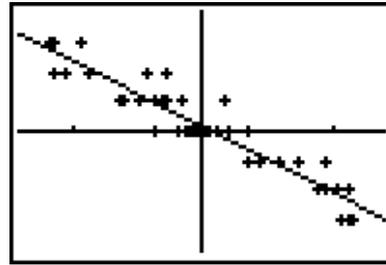


Fig.32.

Conclusion:

Linear character of the observed $U(dB/dt)$ dependence confirms the predictions of the theoretical model and confirm the validity of the Faraday's law of induction.

Expansion:

One can produce separately the time dependence of the voltage (lists L_4 , L_6) and the time dependence of the derivative dB/dt (lists L_4 , L_5). The comparison will show coincidence of those two functions.

Note:

The calculated slope of the linear dependence could be positive when the polarity of the voltage probe leads connections is reversed. Even then, the whole analysis can be performed except the discussion of the Lenz's law.

DATA ANALYSIS (MS EXCEL)

Further analysis can be performed in terms of Faraday's law of induction using tools implemented in calculators and personal computer functions offered by the MS Excel software.

Transfer of experimental data to the personal computer.

After completing the experiment data could be transferred from the graphic calculator to the personal computer.

TI GRAPH LINK™ cable supported by the TI Connect™ software offer tools enabling exploration of the contents of calculator (TI DEVICE EXPLORER) and data edition (TI DATA EDITOR).

Data collected in the experiment are stored in the calculator's lists:

- time in seconds - List L₁
- magnetic induction in mTesla - List L₂
- voltage (induced emf) in Volts - List L₃

Within the TI Connect™ program - the TI DEVICE EXPLORER one can save calculator's lists on the computer's hard disk and then open them within TI DATA EDITOR. Option **Special Lists Export** provides opportunity to save the chosen list as the Excel Comma Separated type file (*.CSV file). Such a file could be open and explore within the MS Excel™ spreadsheet software.

Exemplary data are available for a download in the following files:

- time : as file - Data sample/MSExcel /**time**
- magnetic induction: as file - Data sample/MSExcel/**magnetic induction**
- voltage: as file - Data sample/MSExcel/**voltage**

Analysis of plots

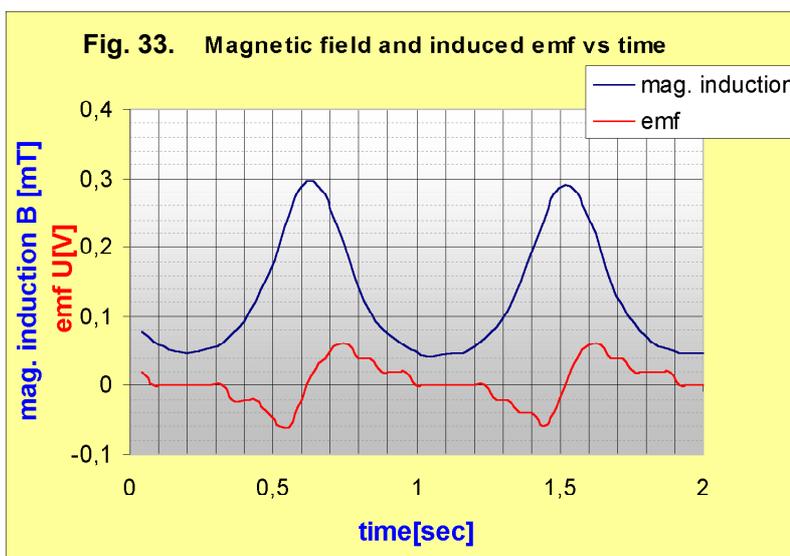
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Fig. 33. presents both $B(t)$ and $emf(t)$ graph.

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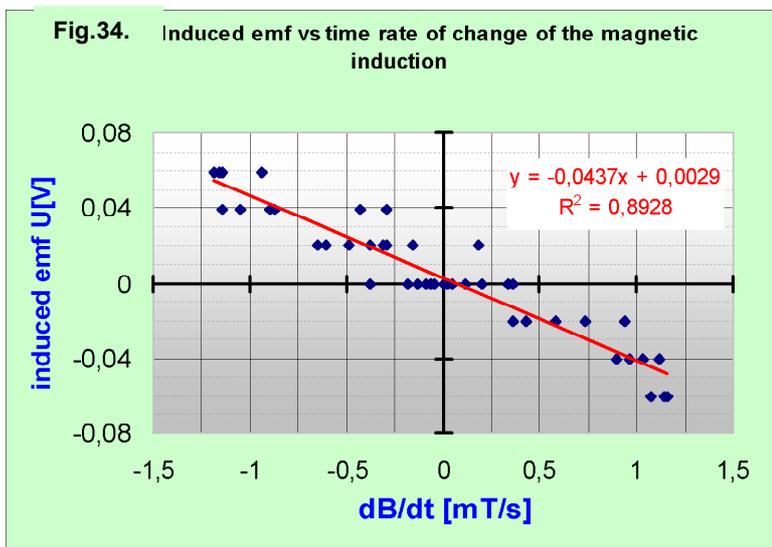
- modified time - column time1- file (available as the separate file: Data sample/MS Excel/**time1**)
- time derivative of the magnetic induction - column dB/dt - (available as the separate file: Data sample/MS Excel/ **time derivative**)
- modified induced emf (voltage) - emf1 - file: voltage1 (available as the separate file: Data sample/MS Excel/ **voltage1**)

Exploration of the Faraday's law calls for a new plot displaying induced emf vs. time derivative of the magnetic induction Fig. 34.

From the obtained plot one can formulate hypothesis of quasi linear dependence between concerned quantities.

This linearity can be examined with the aim of the linear regression function added to the plot.

Slope of the line represents parameters of the coil and can be compared with the theoretical model (see eq.12).



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Linear character of the observed $U(dB/dt)$ dependence confirms the predictions of the theoretical model and confirm the validity of the Faraday's law of induction.

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